DPP 1.1

Subjective Type

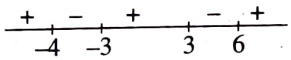
1. $\frac{2x+3}{x^2+x-12} < \frac{1}{2}$

$\Rightarrow \quad \frac{2x+3}{x^2+x-12} - \frac{1}{2} < 0$

$\Rightarrow \quad \frac{4x+6-x^2-x+12}{x^2+x-12} < 0$

$\Rightarrow \quad \frac{x^2-3x-18}{(x+4)(x-3)} > 0$

$\Rightarrow \quad \frac{(x-6)(x+3)}{(x+4)(x-3)} > 0$

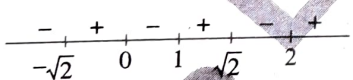


$x \in (-\infty, -4) \cup (-3, 3) \cup (6, \infty)$

2. $\frac{1}{x-2} + \frac{1}{x-1} > \frac{1}{x}$

$\Rightarrow \quad \frac{2x^2-3x-x^2+3x-2}{(x-2)(x-1)x} > 0$

$\Rightarrow \quad \frac{x^2-2}{(x-2)(x-1)x} > 0$



$x \in (-\sqrt{2}, 0) \cup (1, \sqrt{2}) \cup (2, \infty)$

3. $\sqrt{8+2x-x^2} > 6-3x$

We must have $8+2x-x^2 \geq 0$

$\Rightarrow \quad x \in [-2, 4]$ (i)

For $6-2x < 0$ or $x > 2$, inequality is satisfied.

Let $6-3x \geq 0$

$\Rightarrow \quad x \leq 2$ (ii)

$\Rightarrow \quad 8+2x-x^2 > 36+9x^2-36x$

$\Rightarrow \quad 5x^2-19x+14 < 0$

$\Rightarrow \quad (5x-14)(x-1) < 0$

$\Rightarrow \quad x \in \left(1, \frac{14}{5}\right)$ (iii)

From (ii) and (iii), $x \in (1, 2]$

$\therefore$ finally $x \in (1, 4]$

4. We have $\frac{\sqrt{2x-1}}{x-2} < 1$

We must have $x \geq \frac{1}{2}$

Obviously $x < 2$ satisfies the inequality.

For $x > 2$, $\sqrt{2x-1} < x-2$

$\Rightarrow \quad 2x-1 < x^2-4x+4$

$\Rightarrow \quad x^2-6x+5 > 0$

$\Rightarrow \quad x \in (-\infty, 1) \cup (5, \infty)$

From (i), (ii), (iii), $x \in [1/2, 2) \cup (5, \infty)$

5. We have $\sqrt{x-6} - \sqrt{10-x} \geq 1$

We must have $x \geq 6$ and $x \leq 10$

Then $\sqrt{x-6} \geq 1 + \sqrt{10-x}$

Squaring given inequality,

$x-6 \geq 1+10-x+2\sqrt{10-x}$

$\Rightarrow \quad 2x-17 \geq 2\sqrt{10-x}$

Clearly LHS must be positive.

$\therefore \quad x \geq \frac{17}{2}$

So we have

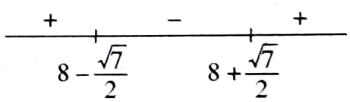
$4x^2+289-68x \geq 4(10-x)$

$\Rightarrow \quad 4x^2-64x+249 \geq 0$

$\Rightarrow \quad x^2-16x+\frac{249}{4} \geq 0$

$\Rightarrow \quad (x-8)^2 - \left(\frac{\sqrt{7}}{2}\right)^2 \geq 0$

$\Rightarrow \quad \left(x-8-\frac{\sqrt{7}}{2}\right)\left(x-8+\frac{\sqrt{7}}{2}\right) \geq 0$



From the sign scheme, $x \in [(16+\sqrt{7})/2, 10]$

6. We have $\frac{1-\sqrt{21-4x-x^2}}{x+1} \geq 0$

We must have $21-4x-x^2 \geq 0$

$\Rightarrow \quad x^2+4x-21 \leq 0$

$\Rightarrow \quad (x+7)(x-3) \leq 0$

$\Rightarrow \quad x \in [-7, 3]$ (i)

Case I: $x+1 > 0 \Rightarrow x > -1$

So given inequality reduces to

$1 \geq \sqrt{21-4x-x^2}$

$\Rightarrow \quad 1 \geq 21-4x-x^2$

$\Rightarrow \quad x^2+4x-20 \geq 0$

$\Rightarrow \quad (x+2)^2 - (2\sqrt{6})^2 \geq 0$

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$\Rightarrow \quad (x+2+2\sqrt{6})(x+2-2\sqrt{6}) \geq 0$

$\Rightarrow \quad x \in [-2+2\sqrt{6}, 3]$

Case II: $x < -1$ (ii)

So given inequality reduces to

$$1-\sqrt{21-4x-x^2} \leq 0$$

$\Rightarrow \quad 1 \leq \sqrt{21-4x-x^2}$

$\Rightarrow \quad 1 \leq 21-4x-x^2$

$\Rightarrow \quad x^2+4x-20 \leq 0$

$\Rightarrow \quad x \in [-2-2\sqrt{6}, -1)$ (iii)

So from (ii) and (iii),

$$x \in [-2-2\sqrt{6}, -1) \cup [-2+2\sqrt{6}, 3]$$

Single Correct Answer Type

7. (b) $f(-1) \geq -4$

$\Rightarrow \quad a-b+c \geq -4$ (i)

$f(1) \leq 0$

$\Rightarrow \quad a+b+c \leq 0$

$\Rightarrow \quad -a-b-c \geq 0$ (ii)

$f(3) \geq 5$

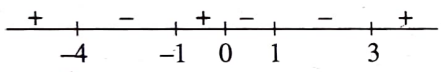
and $9a+3b+c \geq 5$ (iii)

From (i) + (ii) $\Rightarrow -2b \geq -4$ (iv);

From (ii) + (iii) + (iv) $\Rightarrow 8a \geq 1 \Rightarrow a \geq 1/8$

8. (a) We have $E = \dfrac{x^3(x-1)^2(x+4)}{(x+1)(x-3)}$

Sign scheme of E is as shown below.



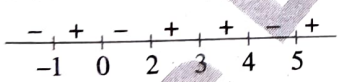
At $x = 1$, sign of the expression does not change as $(x-1)$ has power '2'.

From the sign scheme,

$$x \in (-\infty, -4] \cup (-1, 0] \cup (3, \infty) \cup \{1\}$$

9. (a) We have $E = \dfrac{(x+1)(x-3)^2(x-5)(x-4)^3(x-2)}{x}$

Sign scheme of E is as shown below.



At $x = 3$, sign of the expression does not change as $(x-3)$ has power '2'.

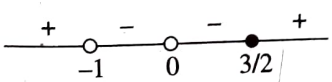
$$x \in (-\infty, -1) \cup (0, 2) \cup (4, 5)$$

10. (b) We have $\dfrac{(e^x-1)(2x-3)(x^2+x+2)}{(\sin x-2)(x+1)x} \leq 0$

$\Rightarrow \quad \dfrac{(e^x-1)(2x-3)}{(x+1)x} \geq 0$

(as $x^2+x+2 > 0$ and $\sin x - 2 < 0$ for all real x)

Sign scheme of E is as shown below.



At $x = 0$, sign of the expression does not change as two factors 'x' and '$e^x - 1$' are becoming zero at this point.

$\therefore$ from the sign scheme, $x \in (-\infty, -1) \cup \left[\dfrac{3}{2}, \infty\right)$.

11. (d) Put $2^x = t$. Then $t > 0$. The given inequality becomes

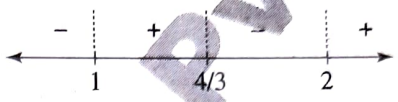
$$\frac{1}{t-1} > \frac{2}{2-t}$$

$\Rightarrow \quad \dfrac{1}{t-1} - \dfrac{2}{2-t} > 0$

$\Rightarrow \quad \dfrac{2-t-2t+2}{(t-1)(2-t)} > 0$

$\Rightarrow \quad \dfrac{4-3t}{(t-1)(2-t)} > 0$

Sign scheme is as shown below.



From sign scheme,

$1 < t < 4/3$ or $t > 2$.

$\Rightarrow \quad 1 < 2^x < 4/3$ or $2^x > 2$

$\Rightarrow \quad 0 < x < \log_2(4/3) \cup (1, \infty)$.

12. (a) $A = (1, 3)$

For $B \subseteq A$: $2^{1-1} + p \leq 0 \Rightarrow p \leq -1$; $2^{1-3} + p \leq 0 \Rightarrow p \leq -1/4$ $\Big\}$ $p \leq -1$

$f(x) = x^2 - 2(p+7)x + 5$

$f(1) \leq 0 \Rightarrow p \geq -4$

$f(3) \leq 0 \Rightarrow P \geq -1$ $\Big\}$ $P \geq -4$

So $p \in [-4, -1]$

13. (b) $a^2+ab+b^2 = \dfrac{a^3-b^3}{a-b} = \dfrac{a^3-b^3}{a^3+b^3} < 1.$

DPP 1.2

Subjective Type

1. $|x|^2 - |x| + 4 = 2x^2 - 3|x| + 1$

$\Rightarrow \quad |x|^2 - |x| + 4 = 2|x|^2 - 3|x| + 1$

$\Rightarrow \quad |x|^2 - 2|x| - 3 = 0$

$\Rightarrow \quad (|x| - 3)(|x| + 1) = 0$

$\Rightarrow \quad |x| - 3 = 0 \Rightarrow |x| = 3 \Rightarrow x = \pm 3$

2. $|x+2| + y = 5$ (i)

$x - |y| = 1$ (ii)

Eliminating x, we get

$|1 + |y| + 2| + y = 5$

$\therefore \quad (3 + |y|) + y = 5$

$\therefore \quad y + |y| = 2$

$\therefore \quad y = 1$

$\therefore \quad x = 2$

3. $x - \sqrt{1-|x|} < 0$

We must have $1 - |x| \geq 0$

$\Rightarrow \quad |x| \leq 1$

$\Rightarrow \quad x \in [-1, 1]$

Case I: $x \in [0, 1]$

So, we have $x < \sqrt{1-x}$

$\Rightarrow \quad x^2 < 1 - x$

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$\Rightarrow \quad x^2 + x - 1 < 0$

$\Rightarrow \quad \left(x + \frac{1}{2}\right)^2 - \left(\frac{\sqrt{5}}{2}\right)^2 < 0$

$\Rightarrow \quad x \in \left[0, -\frac{1}{2} + \frac{\sqrt{5}}{2}\right)$ (i)

Case II: $x \in [-1, 0)$

So, we have $x < \sqrt{1+x}$

$\Rightarrow$ negative < positive

$\Rightarrow \quad x \in [-1, 0)$ (ii)

Hence, from (i) and (ii),

$$x \in \left[-1, -\frac{1}{2} + \frac{\sqrt{5}}{2}\right)$$

4. $||x-1|-2| = |x-3|$

Case I: $x \geq 3$

$\therefore \quad |x-1-2| = x-3 \quad \Rightarrow \quad x-3 = x-3$

$\therefore \quad x \geq 3$

Case II: If $1 \leq x < 3$

$\therefore \quad |x-1-2| = -(x-3)$

$\therefore \quad -(x-3) = -(x-3) \quad \Rightarrow \quad 1 \leq x < 3$

Case III: $-1 \leq x < 1$

$\therefore \quad |-x+1-2| = -(x-3)$

$\therefore \quad |-x-1| = -(x-3) \quad \Rightarrow \quad |x+1| = -x+3$

$x+1 = -x+3 \quad \Rightarrow \quad 2x = 2 \quad \Rightarrow \quad x = 1$

which is not possible.

Case IV: If $x < -1$

$\therefore \quad |-x+1-2| = -(x-3)$

$\Rightarrow \quad |x+1| = -(x-3) \quad \Rightarrow \quad -(x+1) = -(x-3)$

$\Rightarrow \quad -x-1 = -x+3$

$\Rightarrow \quad$ No solution

Hence $x \in [1, \infty)$

5. We have $(|x-1|-3)(|x+2|-5) < 0$

Case I: $x \geq 1$

$\therefore \quad (x-4)(x-3) < 0$

$\therefore \quad x \in (3, 4)$ (i)

Case II: $-2 \leq x < 1$

$\therefore \quad (x-3)(x+2) > 0$

$\therefore \quad x \in (-\infty, -2) \cup (3, \infty)$

$\therefore \quad x \in \phi$ (as $-2 \leq x < 1$)

Case III: $x < -2$

$\therefore \quad (x+2)(x+7) < 0$

$\therefore \quad x \in (-7, -2)$ (ii)

From (i) and (ii), $x \in (-7, -2) \cup (3, 4)$

6. If $a = 0$ or $b = 0$ or $a + b = 0$, then inequality is obvious.

Let $ab(a+b) \neq 0$

$$\frac{|a|}{1+|a|} + \frac{|b|}{1+|b|} \geq \frac{|a|}{1+|a|+|b|} + \frac{|b|}{|a|+1+|b|} \quad \text{(i)}$$

$$= \frac{|a|+|b|}{1+|a|+|b|} = \frac{1}{1+\frac{1}{|a|+|b|}}$$

But $|a+b| \leq |a| + |b|$

$$\Rightarrow \quad \frac{1}{|a+b|} \geq \frac{1}{|a|+|b|}$$

$$\Rightarrow \quad 1 + \frac{1}{|a+b|} \geq 1 + \frac{1}{|a|+|b|}$$

$$\Rightarrow \quad \frac{1}{1+\frac{1}{|a+b|}} \leq \frac{1}{1+\frac{1}{|a|+|b|}} \quad \text{(ii)}$$

From (i) and (ii) $\Rightarrow \quad \frac{|a|}{1+|a|} + \frac{|b|}{1+|b|} \geq \frac{1}{1+\frac{1}{|a+b|}} = \frac{|a+b|}{1+|a+b|}$

Single Correct Answer Type

7. (c) $|2x-3| + |x+5| \leq |x-8|$

$\Rightarrow \quad |2x-3| + |x+5| \leq |(2x-3) - (x-5)|$

$\Rightarrow \quad (2x-3)(x+5) \leq 0 \quad \Rightarrow \quad -5 \leq x \leq 3/2$

8. (c) $|2x+5| - |x-3| \geq |x+8|$

$\Rightarrow \quad |2x+5| - |x-3| \geq |(2x+5) - (x-3)|$

$\Rightarrow \quad (2x+5)(x-3) \geq 0$

$\Rightarrow \quad x \leq -\frac{5}{2}$ or $x \geq 3$ (i)

Also $|2x+5| \geq |x-3|$

Squaring both sides,

$4x^2 + 20x + 25 \geq x^2 - 6x + 9$

$\Rightarrow \quad 3x^2 + 26x + 16 \geq 0$

$\Rightarrow \quad (3x+2)(x+8) \geq 0$

$\Rightarrow \quad x \leq -8$ or $x \geq -\frac{2}{3}$ (ii)

From (i) and (ii), $x \in (-\infty, -8] \cup [3, \infty)$

9. (d) $f(x) = |x - |x-4||$

$$= \begin{cases} 2|x-2|; & x < 4 \\ 4; & x \geq 4 \end{cases}$$

For $|x - |x-4|| = 4$

$2|x-2| = 4 \quad \Rightarrow \quad x = 0, 4$

$\therefore$ solution set is $\{0\} \cup [4, \infty)$.

10. (b) $|2x-3| < |x+2|$

Squaring both sides

$(2x-3)^2 - (x+2)^2 < 0$

$(x-5)(3x-1) < 0$

$\Rightarrow \quad x \in \left(\frac{1}{3}, 5\right)$

11. (d) $\left|\frac{1}{x} - 2\right| < 4 \Leftrightarrow -4 < \frac{1}{x} - 2 < 4 \quad \Leftrightarrow \quad -2 < \frac{1}{x} < 6$

Hence $x \in \left(-\infty, -\frac{1}{2}\right) \cup \left(\frac{1}{6}, \infty\right)$

12. (b) $\left|x + \frac{1}{x}\right| > 2$ (clearly $x \neq 0$)

$\Rightarrow \quad \left|\frac{x^2+1}{x}\right| > 2 \Rightarrow \frac{x^2+1}{|x|} > 2$ ($\because x^2 + 1 > 0$)

$\Rightarrow \quad x^2 + 1 > 2|x|$

$\Rightarrow \quad |x|^2 - 2|x| + 1 > 0 \Rightarrow (|x|-1)^2 > 0$

$\Rightarrow \quad |x| \neq 1 \Rightarrow x \neq -1, 1$

$\therefore \quad x \in R - \{-1, 0, 1\}$

13.(a) $\frac{|x+2| - |x|}{\sqrt{8-x^3}} \geq 0$

We must have $8 - x^3 > 0$

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$\Rightarrow \quad x^3 < 8 \Rightarrow x < 2$

Also $|x+2| - |x| \geq 0$

$\Rightarrow \quad |x+2| \geq |x| \quad \Rightarrow \quad (x+2)^2 - x^2 \geq 0 \quad \Rightarrow \quad 2(x+1) \geq 0$

$\Rightarrow \quad x \geq -1$

So $x \in [-1, 2)$

14. (a) $\left|\dfrac{12x}{4x^2+9}\right| \leq 1$

$$-1 \leq \frac{12x}{4x^2+9} \leq 1$$

Since $4x^2 + 9$ is postive for all real x, we can multiply by $4x^2 + 9$

$\therefore \quad -(4x^2+9) \leq 12x \leq (4x^2+9)$

$(2x+3)^2 \geq 0$ and $(2x-3)^2 \geq 0$

Above is true for all real values of x.

15. (b) $|a-b| = 2 \Rightarrow a - b = \pm 2$

$|b-c| = 3 \Rightarrow b - c = \pm 3$

$|c-d| = 4 \Rightarrow c - d = \pm 4$

Possible values of $a - d$ are $\pm 9, \pm 5, \pm 3, \pm 1$

$\therefore \quad |a-d| = 9, 5, 3, 1$

Sum = 18.

16. (c) $|x| - |x-1| + |x-2| = \sqrt{5}$

For $x \geq 2$,

$x - (x-1) + x - 2 = \sqrt{5} \quad \Rightarrow \quad x = 1 + \sqrt{5}$

For $1 \leq x < 2$,

$x - (x-1) + (2-x) = \sqrt{5}$

$x = 3 - \sqrt{5}$ No solution

For $0 \leq x < 1$,

$x - (1-x) + 2 - x = \sqrt{5}$

$x = \sqrt{5} - 1$ No solution

For $x < 0$,

$-x - (1-x) + 2 - x = \sqrt{5}$

$x = 1 - \sqrt{5}$

Hence, $x = \sqrt{5} + 1$ or $x = 1 - \sqrt{5}$

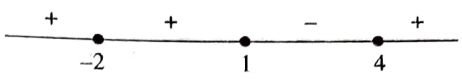
17. (a) $|x^2 - 2x - 8| + |x^2 + x - 2| = 3|x + 2|$

Now $(x^2 + x - 2) - (x^2 - 2x - 8) = 3x + 6 = 3(x + 2)$

$\therefore \quad (x^2 - 2x - 8)(x^2 + x - 2) \leq 0$

$(x-4)(x+2)^2(x-1) \leq 0$

The sign scheme is shown below.



From the sign scheme, $x \in [1, 4] \cup \{-2\}$

18. (b) $|x| + \left|\dfrac{4-x^2}{x}\right| = \left|x + \dfrac{4-x^2}{x}\right|$

As $|a| + |b| = |a+b| \quad \Rightarrow \quad ab \geq 0$

$\therefore \quad x\left(\dfrac{4-x^2}{x}\right) \geq 0, x \neq 0$

$\Rightarrow \quad x^2 - 4 \leq 0$

$\Rightarrow \quad x \in [-2, 2] - \{0\}$

19. (d) We have $|2ax-3| + |ax+1| + |5-ax| = \dfrac{1}{2}$

Now

$|2ax-3| + |ax+1| + |5-ax| \geq |2ax - 3 - (ax+1) + 5 - ax| = 1$

So the given equation has no solution.

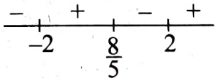
20. (a) $\left|\dfrac{x^2-5x+4}{x^2-4}\right| \leq 1$

$$\Rightarrow \quad -1 \leq \frac{x^2-5x+4}{x^2-4} \leq 1$$

For $\dfrac{x^2-5x+4}{x^2-4} \leq 1$

$\Rightarrow \quad \dfrac{x^2-5x+4}{x^2-4} - 1 \leq 0 \quad \Rightarrow \quad \dfrac{-5x+8}{x^2-4} \leq 0$

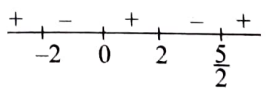
$\Rightarrow \quad \dfrac{5x-8}{(x-2)(x+2)} \geq 0$



$\Rightarrow \quad x \in (-2, 8/5] \cup (2, \infty)$ (i)

Now $\dfrac{x^2-5x+4}{x^2-4} \geq -1$

$\Rightarrow \quad \dfrac{x^2-5x+4}{x^2-4} + 1 \geq 0 \quad \Rightarrow \quad \dfrac{x(2x-5)}{(x-2)(x+2)} \geq 0$



$\Rightarrow \quad x \in (-\infty, -2) \cup [0, 2) \cup [5/2, \infty)$ (ii)

From (i) and (ii), common values are $x \in [0, 8/5] \cup [5/2, \infty)$.

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CHAPTER 2

DPP 2.1

Single Correct Answer Type

1. (b) We have $\sqrt{2x-4} = 1 + \sqrt{x+5}$

Squaring

$2x - 4 = 1 + (x + 5) + 2\sqrt{x+5}$

$\Rightarrow \quad x - 10 = 2\sqrt{x+5}$

$\Rightarrow \quad x^2 + 100 - 20x = 4x + 20$

$\Rightarrow \quad x^2 - 24x + 80 = 0$

$\Rightarrow \quad x = 4, 20$

Putting $x = 4$, we get $\sqrt{4} - \sqrt{9} = 1$, which is not possible

Putting $x = 20$, we get $\sqrt{36} - \sqrt{25} = 1$

Hence, $x = 20$ is the only solution.

2. (b) We have $\sqrt{x} + \sqrt{x - \sqrt{1-x}} = 1$

$\Rightarrow \quad \sqrt{x - \sqrt{1-x}} = 1 - \sqrt{x}$

Squaring,

$x - \sqrt{1-x} = 1 + x - 2\sqrt{x}$

$\Rightarrow \quad 2\sqrt{x} - \sqrt{1-x} = 1$ (i)

$\Rightarrow \quad (2\sqrt{x} - \sqrt{1-x})(2\sqrt{x} + \sqrt{1-x}) = (2\sqrt{x} + \sqrt{1-x})$

$\Rightarrow \quad 4x - (1-x) = 2\sqrt{x} + \sqrt{1-x}$

$\Rightarrow \quad 2\sqrt{x} + \sqrt{1-x} = 5x - 1$ (ii)

Adding (i) and (ii),

$4\sqrt{x} = 5x$

$\Rightarrow \quad 16x = 25x^2$

$\Rightarrow \quad x = 0, \frac{16}{25}$

Clearly $x = 0$ does not satisfy the equation.

Putting $x = \frac{16}{25}$ in equation,

L.H.S. $= \frac{4}{5} + \sqrt{\frac{16}{25} - \frac{3}{5}} = \frac{4}{5} + \frac{1}{5} = 1$

So $x = \frac{16}{25}$ is the only solution.

3. (d) $\frac{2a^2 + x^2}{a^3 - x^3} - \frac{2x}{ax + a^2 + x^2} + \frac{1}{x - a} = 0$

$\Rightarrow \quad \frac{2x^2 - 3ax + a^2}{x^3 - a^3} = 0 \quad (x \neq a)$

$\Rightarrow \quad 2x^2 - 3ax + a^2 = 0$

$\Rightarrow \quad x = a, \frac{a}{2}$

$\therefore \quad \frac{a}{2}$ is the only root if $\frac{a}{2} \neq a$.

$\therefore \quad a \in (-\infty, 0) \cup (0, \infty)$.

4. (c) $x^2 + \left(\frac{x}{x-1}\right)^2 = 8$

$\Rightarrow \quad \left(x + \frac{x}{x-1}\right)^2 - 2x\left(\frac{x}{x-1}\right) = 8$

$\Rightarrow \quad \left(\frac{x^2}{x-1}\right)^2 - 2\left(\frac{x^2}{x-1}\right) - 8 = 0$

$\frac{x^2}{x-1} = t \Rightarrow t^2 - 2t - 8 = 0 \Rightarrow t = 4, t = -2$

$\frac{x^2}{x-1} = 4 \Rightarrow x^2 - 4x + 4 = 0 \Rightarrow x = 2$

Put $\frac{x^2}{x-1} = -2 \Rightarrow x^2 + 2x = 2 \Rightarrow (x+1)^2 = 3$

$\Rightarrow \quad x + 1 = \pm\sqrt{3}$

$\Rightarrow \quad x = \pm\sqrt{3} - 1$

5. (c) We have $m + n\sqrt{2} = \sqrt{41 + 24\sqrt{2}}$

Squaring both sides, we get

$m^2 + 2n^2 + 2\sqrt{2}\,mn = 41 + 24\sqrt{2}$

$\therefore$ On comparing, we get

$m^2 + 2n^2 = 41$ (i)

and $mn = 12$ (ii)

$\therefore$ On solving (i) and (ii), we get

$\Rightarrow \quad m^2 + \frac{2(144)}{m^2} = 41$

$\Rightarrow \quad m^4 - 41m^2 + 288 = 0$

$\Rightarrow \quad (m^2 - 32)(m^2 - 9) = 0$

$\Rightarrow \quad m^2 \neq 32$

$\therefore \quad m^2 = 9 \Rightarrow m = 3$ and $n = 4$

Hence, $(m + n) = 7$

6. (d) Put $(x-1)^{1/2} = t$

or $x = t^2 + 1$

Therefore, the given equation becomes

$(t^2 + 4 - 4t)^{1/2} + (t^2 + 9 - 6t)^{1/2} = 1$

$\Rightarrow \quad [(t-2)^2]^{1/2} + [(t-3)^2]^{1/2} = 1$

$\Rightarrow \quad |t-2| + |t-3| = 1$

This equation is satisfied for all values of t lying between 2 and 3 i.e.,

$2 \leq t \leq 3$

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Thus, the given equation is satisfied for all values of x lying between 5 and 10.

7. (a) We have $\sqrt{3x^2+x+5} = x-3$

We must have $3x^2+x+5 \geq 0$ and $x-3 \geq 0$ or $x \geq 3$

$$\sqrt{3x^2+x+3} = x-3 \quad \text{(i)}$$

Squaring both sides of (i), we get

$3x^2+x+5 = x^2-6x+9$

$\Rightarrow 2x^2+7x-4=0$

$\Rightarrow (2x-1)(x+4)=0$

$\Rightarrow x = 1/2, -4$

These values does not satisfy the inequality $x \geq 3$.

Thus, (i) has no solution.

8. (a) If x is real, $x^2-6|x|+8=0$

$\Rightarrow |x|^2-6|x|+8=0$

$\Rightarrow |x| = 2, 4$

$\Rightarrow x = \pm 2, \pm 4$

If x is non-real, say $x = \alpha + i\beta$, then

$$(\alpha+i\beta)^2 - 6\sqrt{\alpha^2+\beta^2} + 8 = 0$$

$$\therefore \quad \alpha^2-\beta^2+8-6\sqrt{\alpha^2+\beta^2}+2i\alpha\beta = 0$$

Comparing real and imaginary parts,

$\alpha\beta = 0 \Rightarrow \alpha = 0$ (if $\beta = 0$, then x is real)

and $-\beta^2+8-6\sqrt{\beta^2} = 0$

$\therefore \quad \beta^2 \pm 6\beta - 8 = 0$

$$\Rightarrow \beta = \frac{\mp 6 \mp \sqrt{68}}{2}$$

$\Rightarrow \beta = \pm(3-\sqrt{17})$

Hence, $\pm(3-\sqrt{17})i$ are non-real roots.

9. (b) $x^2-(3+2^{\sqrt{\log_2 3}}-3^{\sqrt{\log_3 2}})x-2(3^{\log_3 2}-2^{\log_2 3}) = 0$

$\Rightarrow x^2-3(x)-2(2-3)=0$

$\Rightarrow x^2-3x+2=0$

$\Rightarrow \alpha = 2, \beta = 1$

$\Rightarrow \alpha^2+\alpha\beta+\beta^2 = 4+2+1 = 7$

10. (d) $3x^2 \log 2 - x(1-\log 2) = 2\log 2 - x$ (here base of log is '2')

$3x^2 \log 2 - x + x\log 2 = 2\log 2 - x$

$\therefore \quad 3x^2+x-2=0$

$\therefore \quad (3x-2)(x+1)=0$

$\Rightarrow x = -1, 2/3$

11. (b) Since one root is 3, let

$f(x) = (x-3)(ax+b)$ (1)

$\Rightarrow f(x) = ax^2+(b-3a)x-3b$

Given, $f(-1)+f(2)=0$

$\Rightarrow (a+(3a-b)-3b)+(4a+2b-6a-3b)=0$

$\Rightarrow 2a-5b=0$

$$\Rightarrow \frac{b}{a} = \frac{2}{5} \quad (2)$$

Now, product of roots $= \dfrac{-3b}{a} = -3\left(\dfrac{2}{5}\right)$

Since one root of $f(x)=0$ is 3, the other root $= \dfrac{-2}{5} \in (-3, 0)$

12. (c) According to the question x^2-7x+a is divisible by $x-4$ and x^2+5x+b is divisible by $x+1$.

$\therefore \quad a = 12; b = 4$

13. (b) Put $x^2 = 1$ in the given polynomial

$x+x^2 \cdot x+(x^2)^4 \cdot x+(x^2)^{13} \cdot x+(x^2)^{40} \cdot x+(x^2)^{121} \cdot x = 5x+1$

Comprehension Type

14. (a), 15. (b)

$P(x)+1=0$ has a thrice repeated root at $x=1$.

$P'(x)=0$ has a twice repeated root at $x=1$.

Similarly $P'(x)=0$ has a twice repeated root at $x=-1$.

$\Rightarrow P'(x)$ is divisible by $(x-1)^2(x+1)^2$.

$\because \quad P'(x)$ is of degree at most 4.

$\Rightarrow P'(x) = \alpha(x-1)^2(x+1)^2$

$$\therefore \quad P(x) = \alpha\left(\frac{x^5}{5}-\frac{2}{3}x^3+x\right)+c$$

Now $P(1) = -1$ and $P(-1) = 1$

$$\Rightarrow \alpha = -\frac{15}{8} \text{ and } c = 0$$

$$\therefore \quad P(x) = -\frac{3}{8}x^5+\frac{5}{4}x^3-\frac{15}{8}x$$

16. (c), 17. (b)

$f(x) = ax^2+bx+c,\ a \neq 0,\ a, b, c \in I$

$f(1) = 0$

$\Rightarrow a+b+c=0$ (i)

$50 < f(7) < 60$

$50 < 49a+7b+c < 60$

$\Rightarrow 50 < 48a+6b < 60$

$$\Rightarrow \frac{50}{6} < 8a+b < 10$$

$\Rightarrow 8a+b=9$ (ii)

Also $70 < f(8) < 80$

$\Rightarrow 70 < 64a+8b+c < 80$

$\Rightarrow 70 < 63a+7b < 80$

$$\Rightarrow 10 < 9a+b < \frac{80}{7}$$

$\Rightarrow 9a+b=11$ (iii)

From (i), (ii) and (iii), $a=2, b=-7, c=5$

$\Rightarrow f(x) = 2x^2-7x+5 = (2x-5)(x-1)$

$$= 2\left(x^2-\frac{7}{2}x+\frac{5}{2}\right)$$

$$= 2\left(\left(x-\frac{7}{4}\right)^2-\frac{9}{16}\right)$$

$\Rightarrow f(x)$ has least value $-\dfrac{9}{8}$

$f(x) < 0 \Rightarrow (2x-5)(x-1) < 0 \Rightarrow 1 < x < 5/2$

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DPP 2.2

Single Correct Answer Type

1. (a) a = sum of the roots = odd
Therefore, both the roots cannot be odd.
Therefore, one root is 2.
$\therefore \quad 4 - 2a + b = 0$ (i)
Given $a + b = 35$ (ii)
by solving (1) and (2) $a = 13$, $b = 22$
$\therefore \quad f(x) = x^2 - 13x + 22$
$\therefore \quad f(10) = -8$
$\therefore$ Required value = 880

2. (a) Let $f(x) = (x - \sin\beta)(x - \sin\gamma) + (x - \sin\alpha)(x - \sin\gamma) + (x - \sin\alpha)(x - \sin\beta)$
Now, $f(\sin\alpha) = (\sin\alpha - \sin\beta)(\sin\alpha - \sin\gamma)$
$= (-)(-) =$ positive
$f(\sin\beta) = (\sin\beta - \sin\alpha)(\sin\beta - \sin\gamma) = (+)(-) =$ negative
$f(\sin\gamma) = (\sin\gamma - \sin\alpha)(\sin\gamma - \sin\beta) = (+)(+) =$ positive
$\Rightarrow$ Roots of $f(x) = 0$ are real and distinct.

3. (b) Eliminating t, we get

$$r + s + r^2 + s^2 = \frac{k-3}{2}$$

or $$r^2 + r + s^2 + s + \frac{3-k}{2} = 0$$

Now for one real solution 'r'
$D = 0$

$$\therefore \quad 1 - 4\left(s^2 + s + \frac{3-k}{2}\right) = 0$$

$$\therefore \quad 4s^2 + 4s + 5 - 2k = 0$$

Now for one real solution 's'
$D = 0$
$\therefore \quad 16 - 16(5 - 2k) = 0$
$\therefore \quad 1 = 5 - 2k$
$\therefore \quad k = 2$

4. (c) $f(x) = ax^4 + bx^3 + cx^2 + 5x - 7$
$f'(x) = 4ax^3 + 3bx^2 + 2cx + 5$
$f''(x) = 12ax^2 + 6bx + 2c = 0$
$\Rightarrow$ disc. $= 36b^2 - 4(12a)(2c)$
$= 12(3b^2 - 8ac) < 0$
$(\because 3b^2 < 8ac)$
$f'(x) = 0$ has only one real root
$\Rightarrow f(x) = 0$ has exactly two real roots.

5. (b) $\sqrt[3]{x + 3p + 1} = \sqrt[3]{x} + 1$
Let $\sqrt[3]{x} = h$
$\Rightarrow \quad \sqrt[3]{x + 3p + 1} = h + 1$
$\Rightarrow \quad x + 3p + 1 = h^3 + 3h^2 + 3h + 1$
$\Rightarrow \quad h^3 + 3p + 1 = h^3 + 3h^2 + 3h + 1$
$\Rightarrow \quad 3h^2 + 3h - 3p = 0$
$\Rightarrow \quad h^2 + h - p = 0$
For real roots $D \geq 0$
$\Rightarrow \quad 1 + 4p \geq 0$
$\Rightarrow \quad p \geq -1/4$

6. (c) $(a^2 + b^2)x^2 - 2b(a + c)x + b^2 + c^2 = 0$
$D = 4b^2(a + c)^2 - 4(a^2 + b^2)(b^2 + c^2)$
$= -4(b^4 - 2b^2ac + a^2c^2)$
$= -4(b^2 - ac)^2$
For real roots, $D \geq 0$
$\Rightarrow \quad -4(b^2 - ac)^2 \geq 0$
$\Rightarrow \quad b^2 - ac = 0$
$\Rightarrow$ Roots are real and equal.
$\therefore$ Roots are $\dfrac{2b(a+c) \pm 0}{2(a^2 + b^2)}$

$$= \frac{b(a+c)}{a^2 + ac}$$

$$= \frac{b}{a}$$

This root satisfies option (c).

7. (d) Let r_1, r_2 be the roots, from given condition

$$r - 2 = \frac{1}{r} \Rightarrow r^2 - 2r - 1 = 0,\ b = -2,\ c = -1,\ b^2 + c^2 = 5.$$

8. (d) $f(x) = x^2 + 3x + 4$
$\therefore \quad f(x) = x$ has no real solution
$\therefore \quad f(f(x)) = f(x) = x$ also has no real solution.

Multiple Correct Answers Type

9. (a, b)

We have $$\frac{p}{2x} = \frac{(a+b)x + c(b-a)}{x^2 - c^2}$$

$$\Rightarrow \quad p(x^2 - c^2) = 2(a+b)x^2 - 2c(a-b)x$$

$$\Rightarrow \quad (2a + 2b - p)x^2 - 2c(a-b)x + pc^2 = 0$$

Since roots are equal

$$D = c^2(a-b)^2 - pc^2(2a + 2b - p) = 0$$

$$\Rightarrow \quad (a-b)^2 - 2p(a+b) + p^2 = 0 \quad (\because c^2 \neq 0)$$

$$\Rightarrow \quad [p - (a+b)]^2 = (a+b)^2 - (a-b)^2$$

$$\Rightarrow \quad p = a + b \pm 2\sqrt{ab} = (\sqrt{a} \pm \sqrt{b})^2$$

10. (a, c)

$$(ay - bx)^2 + 4xy = 0$$

$$\Rightarrow \quad a^2y^2 + b^2x^2 + (4 - 2ab)xy = 0$$

$$\Rightarrow \quad \frac{a^2y}{x} + \frac{b^2x}{y} - (2ab - a) = 0$$

Put $\dfrac{y}{x} = t$

$$\Rightarrow \quad a^2t^2 - (2ab - 4)t + b^2 = 0$$

$$D = (2ab - 4)^2 - 4a^2b^2$$

$= 4(4 - 4ab) = 16(1 - ab)$ must be a perfect square.

11. (b, c, d)

If at least one of the equations has real roots, then

$$D_1 + D_2 + D_3 \geq 0$$

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$(b^2 - 4acm) + (c^2 - 4bam) + a^2 - 4cbm \geq 0$

$a^2 + b^2 + c^2 \geq 4(ab + bc + ca)m$

$4m \leq \dfrac{a^2+b^2+c^2}{ab+bc+ca}$ (1) $\forall$ $a, b, c \in R+$

but $a^2 + b^2 \geq 2ab$ etc.

$\therefore$ $a^2 + b^2 + c^2 \geq ab + bc + ca$

$\dfrac{a^2+b^2+c^2}{ab+bc+ca} \geq 1$

$\therefore$ $\left.\dfrac{a^2+b^2+c^2}{ab+bc+ca}\right|_{\min} = 1$; Hence $4m$ must be less than or equal to the minimum value.

$\therefore$ $4m \leq 1$

$\Rightarrow$ $m \leq \dfrac{1}{4}$

$\Rightarrow$ $m \in \left(0, \dfrac{1}{4}\right]$

DPP 2.3

Single Correct Answer Type

1. (c) $-x^2 + 8x - 7 = m \Rightarrow x^2 - 8x + 7 + m = 0$

$\alpha + 3\alpha = 8 \Rightarrow \alpha = 2$

$\alpha \cdot 3\alpha = 7 + m \Rightarrow 12 = 7 + m \Rightarrow m = 5$

2. (c) Sum = Product

$\dfrac{4}{a} = a \Rightarrow a^2 = 4$

Also $D < 0 \Rightarrow (4)^2 - 4a^3 < 0$

$4(4 - a^3) < 0$

$a = 2$

3. (c) Let p, q are roots.

$p^2 + q^2 = (p + q)^2 - 2pq$

$= (\sin\alpha - 2)^2 + 2(1 + \sin\alpha)$

$= (\sin\alpha - 1)^2 + 5$

For least value $\sin\alpha = 1$

$\Rightarrow$ $\alpha = \pi/2$

4. (d) $\alpha + \beta = -\dfrac{b}{a}$; $\alpha\beta = \dfrac{c}{a}$

Sum of roots of the equation $a^2x^2 + (b^2 - 2ac)x + b^2 - 4ac = 0$

$x_1 + x_2 = \dfrac{2ac - b^2}{a^2}$

$= \dfrac{2c}{a} - \left(\dfrac{b}{a}\right)^2$

$= 2\alpha\beta - (\alpha + \beta)^2$

$= -(\alpha^2 + \beta^2)$

5. (b) Roots of $ax^2 + bx - b = 0$ are α and β.

$\therefore$ $\alpha + \beta = \dfrac{-b}{a}$ and $\alpha\beta = \dfrac{-b}{a}$

$\therefore$ $\alpha + \beta - \alpha \cdot \beta = 0$

$\therefore$ $(\alpha - 1)(\beta - 1) = 1$

$\therefore$ $|\alpha - 1| \cdot |\beta - 1| = 1$

$\therefore$ $\log_{|(\beta-1)|}|(\alpha - 1)| = -1$

6. (c)

For the 1st equation $(\cos^4\alpha + k) - (\sin^4\alpha + k) = \sqrt{4\lambda^2 - 4\lambda}$

i.e., $\cos^4\alpha - \sin^4\alpha = \sqrt{4\lambda^2 - 4\lambda}$

For the 2nd equation $\cos^2\alpha - \sin^2\alpha = \sqrt{64 - 16} = \sqrt{48}$

But $\cos^4\alpha - \sin^4\alpha = (\cos^2\alpha - \sin^2\alpha)(\cos^2\alpha + \sin^2\alpha)$

$= \cos^2\alpha - \sin^2\alpha$

$\Rightarrow$ $4\lambda^2 - 4\lambda = 48$

$\Rightarrow$ $\lambda = 4$ or -3

Sum = 1

7. (c) $|\alpha - \beta|^2 = |(\alpha + \delta) - (\beta + \delta)|^2$

$\Rightarrow$ $(\alpha + \beta)^2 - 4\alpha\beta$

$= ((\alpha + \delta) + (\beta + \delta))^2 - 4(\alpha + \delta)(\beta + \delta)$

$\Rightarrow$ $\dfrac{b^2}{a^2} - \dfrac{4c}{a} = \dfrac{q^2}{a^2} - \dfrac{4r}{a}$

$\Rightarrow$ $\dfrac{b^2 - 4ac}{4a} = \dfrac{q^2 - 4ar}{4a}$

$\Rightarrow$ $f_{\max} = -\dfrac{D}{4a} = g_{\max}$

8. (c) Because $b^2 = ac$, the roots of both the equations are equal.

$\therefore$ $\alpha = \beta$ and $\gamma = \delta$.

But $\gamma = \dfrac{1}{\alpha}$ and $\delta = \dfrac{1}{\beta}$ as the given equations are reciprocal to each other.

$\therefore$ $\gamma\delta = \dfrac{a}{c}$, then $c\gamma = a\beta$

$\therefore$ $a\alpha = a\beta = c\gamma = c\delta$

9. (a) $S_{n+1} = \alpha^{n+1} + \beta^{n+1}$

$= (\alpha + \beta)(\alpha^n + \beta^n) - \alpha\beta(\alpha^{n-1} + \beta^{n-1})$

$= -\dfrac{b}{a} \cdot S_n - \dfrac{c}{a} \cdot S_{n-1}$

10. (a) $a\alpha^2 + b\alpha + c = a\alpha^2 + p\alpha + q = 0$

$\Rightarrow$ $\alpha = \dfrac{q - c}{b - p}$ (i)

and $b^2 - 4ac = p^2 - 4aq$

$\Rightarrow$ $b^2 - p^2 = 4a(c - q)$

$\Rightarrow$ $b + p = \dfrac{4a(c - q)}{b - p} = -4a\alpha$ (from (i))

$\therefore$ $\alpha = \dfrac{-(b + p)}{4a} = \dfrac{\frac{-b}{a} - \frac{p}{a}}{4}$ which is AM of all the roots of $f(x) = 0$ and $g(x) = 0$.

11. (a) $\alpha + \beta = -3$; $\alpha\beta = 1$.

Also $\alpha^2 + 3\alpha + 1 = 0$ and $\beta^2 + 3\beta + 1 = 0$

$\Rightarrow$ where $\alpha^2 = -(3\alpha + 1)$ and $\beta^2 = -(3\beta + 1)$

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$$y = \frac{\alpha^2}{(1+\beta)^2} + \frac{\beta^2}{(\alpha+1)^2}$$
$$= \frac{\alpha^2}{1+2\beta+\beta^2} + \frac{\beta^2}{1+2\alpha+\alpha^2}$$
$$= \left(\frac{-(3\alpha+1)}{-\beta}\right) + \left(\frac{-(1+3\beta)}{-\alpha}\right)$$
$$= \frac{1+3\alpha}{\beta} + \frac{1+3\beta}{\alpha}$$
$$= \frac{\alpha(1+3\alpha)+\beta(1+3\beta)}{\alpha\beta}$$
$$= 3(\alpha^2+\beta^2) + (\alpha+\beta) \quad (\text{as } \alpha\beta = 1)$$
$$= 3[9-2] + (-3) = 21 - 3 = 18$$

12. (a) As it is given that $ab + bc + ca = 0$, so putting $x = 1$ in the equation, we get

$f(1) = a(b-2c) + b(c-2a) + c(a-2b)$

$\Rightarrow \quad f(1) = -(ab+bc+ac) = 0$

So, 1 is a root of the equation.

Now product of the roots be 1.

$$\therefore \quad 1 \cdot \alpha = \frac{c(a-2b)}{a(b-2c)} \Rightarrow \alpha = \frac{c}{a}\left(\frac{a-2b}{b-2c}\right)$$

Comprehension Type

13. (a), 14. (b), 15. (d)

$$\alpha^2 + \beta^2 = 5 \quad \text{(i)}$$
$$3(\alpha^5+\beta^5) = 11(\alpha^3+\beta^3) \quad \text{(ii)}$$
$$\frac{\alpha^5+\beta^5}{\alpha^3+\beta^3} = \frac{11}{3}$$
$$\therefore \quad \frac{(\alpha^3+\beta^3)(\alpha^2+\beta^2) - (\alpha^2\beta^2(\alpha+\beta))}{\alpha^3+\beta^3} = \frac{11}{3}$$
$$\therefore \quad \alpha^2+\beta^2 - \frac{\alpha^2\beta^2(\alpha+\beta)}{(\alpha+\beta)(\alpha^2+\beta^2-\alpha\beta)} = \frac{11}{3}$$
$$\therefore \quad 5 - \frac{\alpha^2\beta^2}{5-\alpha\beta} = \frac{11}{3}$$
$$\therefore \quad \frac{25-5\alpha\beta-\alpha^2\beta^2}{5-\alpha\beta} = \frac{11}{3}$$

Let $\alpha\beta = t$

$$\frac{25-5t-t^2}{5-t} = \frac{11}{3}$$
$$75 - 15t - 3t^2 = 55 - 11t$$
$$75 - 15t - 3t^2 - 55 + 11t = 0$$
$$-3t^2 - 4t + 20 = 0$$
$$(t-2)(3t+10) = 0$$
$$\therefore \quad t = 2 \text{ or } \frac{-10}{3}$$

So $\alpha\beta = 2, \alpha\beta = \frac{-10}{3}$

If $\alpha\beta = 2$,

$$\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$$
$$\therefore \quad 5 = (\alpha+\beta)^2 - 2\times 2$$
$$(\alpha+\beta)^2 = 9$$
$$\alpha+\beta = \pm 3$$

for $\alpha\beta = \frac{-10}{3}$, $(\alpha+\beta)^2 < 0$

$\Rightarrow \quad x^2 \pm 3x + 2 = 0$

DPP 2.4

Single Correct Answer Type

1. (c) First equation has irrational roots.

Therefore, both roots are common

2. (a) α is root of all equations.

$$\therefore \quad a\alpha^2 + 2b\alpha + c = 0$$
$$2b\alpha^2 + c\alpha + a = 0$$
$$c\alpha^2 + a\alpha + 2b = 0$$

Adding $(a+2b+c)(1+\alpha+\alpha^2) = 0$

As $a, b, c \in R^+$, $\alpha + \alpha^2 = -1$

3. (d) Let α be the common root.

$$\therefore \quad \alpha^4 + 2\alpha^3 - 8\alpha^2 - 6\alpha + 15 = 0$$
$$\alpha^3 + 4\alpha^2 - \alpha - 10 = 0$$

$\therefore$ Given equations are

$(\alpha^2-3)(\alpha^2+2\alpha-5) = 0$ and $(\alpha+2)(\alpha^2+2\alpha-5) = 0$

$\Rightarrow$ For common roots, $\alpha^2 + 2\alpha - 5 = 0$

$\therefore$ Product of uncommon real roots is $(-3)(-2) = 6$.

4. (d) Given equations are

$$x^2 - px + 20 = 0$$
$$x^2 - 20x + p = 0$$

If $p = 20$ both the quadratic equations are identical hence $x = 10 + 4\sqrt{5}$ or $x = 10 - 4\sqrt{5}$

If $p \neq 20$, then subtracting equations, we get

$$(20-p)x + (20-p) = 0$$

$\Rightarrow \quad x = -1$

Hence there are 3 values of x

5. (c) Roots of $4x^2 - x - 1 = 0$ are irrational.

So, one root common implies both roots are common.

$$\therefore \quad \frac{4}{3} = \frac{-1}{\lambda+\mu} = \frac{-1}{\lambda-\mu} = \frac{-2}{2\lambda} = \frac{0}{2\mu}.$$

So, $\lambda = \frac{-3}{4}$, $\mu = 0$

6. (a) $(x+\lambda)^2 + 1 = 0$ has clearly imaginary roots

So, both roots of the equations are common

$$\therefore \quad \frac{a}{1} = \frac{b}{2\lambda} = \frac{c}{\lambda^2+1} = k \text{ (say)}$$

Then $a = k$, $b = 2\lambda k$, $c = (\lambda^2+1)k$

As a, b, c are sides of triangle

$$a + b > c \Rightarrow 2\lambda + 1 > \lambda^2 + 1 \Rightarrow \lambda^2 - 2\lambda < 0$$

$\Rightarrow \quad \lambda \in (0, 2)$

The other conditions also imply same relation.

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7. (b) Given equation can be written as

$(6k+2)x^2 + rx + 3k - 1 = 0$ (i)

and $2(6k+2)x^2 + px + 2(3k-1) = 0$ (ii)

Condition for common roots is

$$\frac{12k+4}{6k+2} = \frac{p}{r} = \frac{6k-2}{3k-1} = 2 \text{ or } 2r - p = 0$$

8. (b) Let the roots of $x^3 + 5x^2 + px + q = 0$ are $\alpha_1, \beta_1, \gamma_1$ (i)

Then roots of the $x^3 + 7x^2 + px + r = 0$ is $\alpha_1, \beta_1, \gamma_2$ (ii)

From (i) – (ii), $-2x^2 + q - r = 0$

This equation has roots α_1, β_1

$\Rightarrow \alpha_1 + \beta_1 = 0$

Now from (i) $\alpha_1 + \beta_1 + \gamma_1 = -5 \Rightarrow \gamma_1 = -5$

From (ii) $\alpha_1 + \beta_1 + \gamma_2 = -7 \Rightarrow \gamma_2 = -7$

$\therefore \gamma_1 + \gamma_2 = -12$

$\therefore |\gamma_1 + \gamma_2| = 12$

9. (c) $a > 1, b > 1, a \neq b$.

Let α is the common root

$(a-1)\alpha^2 - (a^2+2)\alpha + a^2 + 2a = 0$ (i)

$(b-1)\alpha^2 - (b^2+2)\alpha + (b^2+2b) = 0$ (ii)

(i) $\times (b-1)$ – (ii) $\times (a-1)$, we get

$(a-b)(ab-a-b-2)(\alpha - 1) = 0$ (iii)

If $\alpha = 1$, then $a = b = 1$

So, $a \neq 1$

From (iii), we get

$ab = a + b + 2$

Let $a > b > 1$

$$\therefore b = 1 + \frac{b}{a} + \frac{2}{a} < 3 \Rightarrow b = 2, a = 4$$

$\therefore ab = 8$

10. (c) Let $p(x) = x^2 + ax + b$ and let α be the common root

$\therefore p(\alpha) = 0$ and $p(p(p(\alpha))) = 0 \Rightarrow p(p(0)) = 0$

$\Rightarrow p(\alpha) = 0$

Now $p(0) = b$

$\therefore p(b) = 0$

$\Rightarrow b^2 + ab + b = 0$

$\Rightarrow b(b + a + 1) = 0$

$\Rightarrow p(0)\, p(1) = 0.$

11. (d) $D_1 = b^2 - 4ac < 0$, $D_2 = b^2 - 4ac < 0$, as the root is non-real

$\Rightarrow$ Both roots will be common.

$$\Rightarrow \frac{a}{c} = \frac{b}{b} = \frac{c}{a} = 1 \Rightarrow a = c$$

Now, $b^2 - 4ac < 0 \Rightarrow b^2 - 4a^2$ (or $4c^2$) < 0

$\Rightarrow |b| < 2|a|$ (or $2|c|$).

12. (c) **Case I:** when Q is a quadratic equation

$D_P = (m+3)^2$ and $D_Q = (m-2)^2$

roots of 1st equation are $2, -(m+1)$

roots of 2nd equation are $-1, \frac{1}{1-m}$,

For exactly there elements in $P \cup Q$ two of the roots must be same

So we have following possibilities

$2 = -(m+1) \quad \Rightarrow m = -3$

$$2 = \frac{1}{1-m} \quad \Rightarrow m = 1/2$$

$-m - 1 = -1 \quad \Rightarrow m = 0$

$$-(m+1) = \frac{1}{1-m} \Rightarrow 1 - m^2 = -1 \Rightarrow m = \pm\sqrt{2}$$

$$\frac{1}{1-m} = -1 \quad \Rightarrow m = 2.$$

Case II: Now if $m = 1$, then Q becomes linear

roots of Q as $x = -1$

roots of P are 2 and –2

$\Rightarrow$ 3 elements in common

$\therefore$ all permissible values of m are $\left\{-3, \frac{1}{2}, \sqrt{2}, -\sqrt{2}, 2, 0, 1\right\}$

Comprehension Type

13. (a)

$x^2 - ax + b = 0$ (i)

$x^2 - px + q = 0$ (ii)

Let the roots of (i) be α, β and that of (ii) be $\alpha, \frac{1}{\beta}$

$\therefore \alpha + \beta = a, \alpha\beta = b; \alpha + \frac{1}{\beta} = p, \frac{\alpha}{\beta} = q.$

$$\therefore \frac{(q-b)^2}{(p-a)^2} = \frac{\alpha^2\left(\frac{1}{\beta} - \beta\right)^2}{\left(\frac{1}{\beta} - \beta\right)^2}$$

But $\alpha^2 = \alpha\beta \cdot \frac{\alpha}{\beta} = bq$

$\Rightarrow (q-b)^2 = bq(p-a)^2$

14. (c)

Let α, β be the roots of (i) and α, α those of (ii)

$\therefore \alpha + \beta = a, \alpha\beta = b; 2\alpha = p, \alpha^2 = q$

$$\therefore b + q = \alpha(\beta + \alpha) = \frac{1}{2}p.a$$

15. (c)

$\alpha + \beta = a; \alpha\beta = b; \gamma + \delta = p, \gamma\delta = q$

$\alpha, \beta, \gamma, \delta$ are in A.P., $\beta - \alpha = \delta - \gamma$

$\Rightarrow (\beta + \alpha)^2 - 4\alpha\beta = (\delta + \gamma)^2 - 4\gamma\delta$

$\Rightarrow a^2 - 4b = p^2 - 4q$

$$\Rightarrow q - b = \frac{p^2 - a^2}{4}$$

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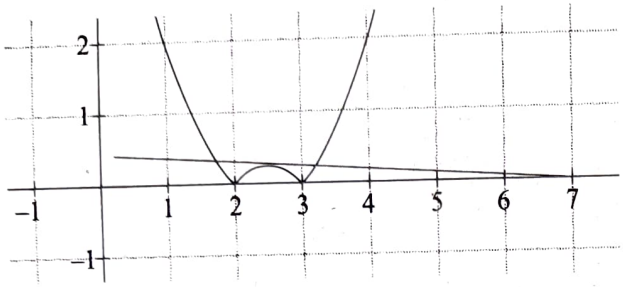
DPP 2.5

Single Correct Answer Type

1. (b) $|x^2 - 5x + 6| - \lambda x + 7\lambda = 0$ has exactly three solutions,

$\therefore$ as shown in the graph

$-x^2 + 5x - 6 = \lambda x - 7\lambda$ must have equal roots



$\therefore \quad x^2 + (\lambda - 5)x + 6 - 7\lambda = 0$ has discriminant zero,

$\Rightarrow \quad \lambda = -9 + 4\sqrt{5}$

2. (c) According to the given condition, we have

$|am^2 + bm + c| = am^2 + bm + c$

i.e. $am^2 + bm + c > 0$

$\Rightarrow$ if $a < 0$, then m lies in (α, β)

and if $a > 0$, then m does not lie in (α, β)

Hence, option (c) is correct, since

$\frac{|a|}{a} = 1 \Rightarrow a > 0$ and in that case m does not lie in (α, β).

3. (a) $D_1 = 4b_1^2 - 4a_1c_1 < 0$ and $a_1 > 0$

i.e. $a_1c_1 > b_1^2$ (i)

$D_2 = 4b_2^2 - 4a_2c_2 < 0$ and $a_2 > 0$

Hence $a_2c_2 > b_2^2$ and $a_2 > 0$ (ii)

Multiplying (i) and (ii), we get

$a_1a_2c_1c_2 > b_1^2b_2^2$

Now consider $g(x)$

$D = b_1^2b_2^2 - 4a_1a_2c_1c_2 < b_1^2b_2^2 - 4b_1^2b_2^2$

$= -3b_1^2b_2^2$

$\therefore \quad D < 0$

$\Rightarrow \quad g(x) > 0 \ \forall \ x \in R \quad (\because a_1, a_2 > 0)$

4. (b)

	(a)	(b)	(c)	(d)
a (using concavity)	>0	<0	<0	>0
c (using f(0))	>0	<0	>0	<0
b (using abscissa of vertex $(-b/2a)$)	>0	<0	<0	<0
abc	>0	<0	>0	>0

5. (d) Let $\frac{x^2 + 4x + 9}{x^2 + 9} = y$

$\Rightarrow \quad (y - 1)x^2 - 4x + 9(y - 1) = 0$

For real value of x, $D \geq 0$

$(5 - 3y)(3y - 1) \geq 0$

$\Rightarrow \quad \frac{1}{3} \leq y \leq \frac{5}{3}$

$\therefore$ Difference $= \frac{5}{3} - \frac{1}{3} = \frac{4}{3}$

6. (c) Sum of roots $= \frac{3a}{a-1} \Rightarrow$ +ve

Product of roots $= \frac{1}{a-1} \Rightarrow$ +ve

Also $D = 9a^2 + 4(1 - a) = a(9a - 4) + 4 > 0$

Hence, both positive roots.

7. (b) Let $f(x) = ax^2 + (a - 2)x - 2$

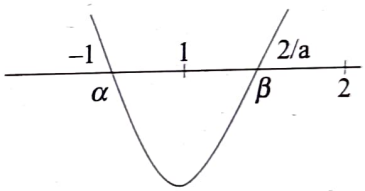
$f(x)$ is negative for two integral values of x, so graph should be vertically upward parabola

i.e., $a > 0$

Let two roots of $f(x) = 0$ are α and β, then

$\alpha, \beta = \frac{-(a-2) \pm (a+2)}{2a}$

$\Rightarrow \quad \alpha = -1, \beta = \frac{2}{a}$



$\Rightarrow \quad 1 < \beta \leq 2$

$\Rightarrow \quad 1 < \frac{2}{a} \leq 2$

$\Rightarrow \quad a \in [1, 2)$

8. (c) $(a + 1)x^2 - 3ax + 4a = 0$

$D = 9a^2 - 16a(a + 1) \geq 0$ (i)

$x_1 > 1, x_2 > 1$

$\therefore \quad (a + 1)f(1) > 0$ and $\frac{3a}{2(a+1)} > 1$

$\Rightarrow \quad (a + 1)(2a + 1) > 0$

and $\frac{a-2}{a+1} > 0$ (ii)

$\frac{2a+1}{a+1} > 0$ (iii)

Solving (i), (ii) and (iii), we get $-\frac{16}{7} \leq a < -1$.

9. (a) Putting $x^2 = y$, the given equation in x reduces to

$ay^2 - 2y - (a - 1) = 0$ (i)

The given biquadratic equation will have four real and distinct roots, if the quadratic equation (i) has two distinct and positive roots.

For that, we must have

$D > 0 \Rightarrow a^2 - a + 1 > 0$, which is true $\forall \ a \in R$

Product of roots $> 0 \Rightarrow 0 < a < 1$

Sum of roots $> 0 \Rightarrow a > 0$

Hence, the acceptable values of a are $0 < a < 1$.

10. (a) Let x_1 and x_2 be two roots of $ax^2 + bx + c = 0$

$1 < x_1 < 2$ and $1 < x_2 < 2$

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Now, $a(5a+2b+c) = a^2\left(5+2\frac{b}{a}+\frac{c}{a}\right)$

$= a^2(5+2(-1)(x_1+x_2)+x_1x_2)$

$= a^2\{(x_1-2)(x_2-2)+1\} > 0$

Hence, a and $5a+2b+c$ are of same sign.

11. (c) $bx^2+(1-b(c+d))x+bcd-a=0$

$\Rightarrow bx^2+x-bcx-bdx+bcd-a=0;$

$\Rightarrow bx(x-c)-bd(x-c)+x-a=0$

$\Rightarrow b(x-c)(x-d)+(x-a)=0$

Let $f(x)=b(x-c)(x-d)+(x-a)$

$\therefore f(c)=c-a<0; f(d)=d-a>0$

12. (b) Since $2a, b, 2c$ are in A.P., therefore

$2a+2c=2b$

$\Rightarrow a-b+c=0$

$\therefore f(1)=0$

Also $f(0)=c>0$

Therefore, product of roots is positive.

Therefore, other root is also positive.

13. (a) Let $f(x)=(a+b-2c)x^2+(b+c-2a)x+(c+a-2b)$

According to the given condition, we have

$f(0)f(-1)<0$

$\Rightarrow (c+a-2b)(2a-b-c)<0$

$\Rightarrow (c+a-2b)(a-b+a-c)<0$

$\Rightarrow c+a-2b<0$

$[a>b>c$, given $\Rightarrow a-b>0, a-c>0]$

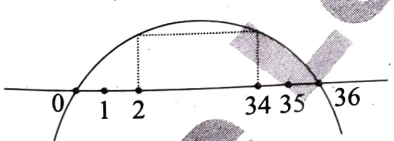
$\Rightarrow b>\frac{a+c}{2}$

$\Rightarrow b$ cannot be the G.M. of a, c, since G.M < A.M. always.

14. (a) $x^2-36x+\lambda=0$

$\therefore \lambda=36x-x^2$

$\alpha=1, \lambda=35, \beta=35$ (rejected)



$\therefore \alpha=2, \lambda=68, \beta=34$ or $\alpha=34, \beta=2$

$\therefore \frac{\sqrt{36}\sqrt{4}}{32}=\frac{3}{8}$

15. (b) Put $2^x=t$

$\Rightarrow t^2+2a\cdot t+(a+1)=0$

Now, $t=1$ should lie between the roots of above equation.

Let $f(t)=t^2+2a\cdot t+(a+1)$

$\therefore f(1)<0$ and $f(0)>0$

$\Rightarrow a<\frac{-2}{3}$ and $a>-1$

$\therefore a\in\left(-1,\frac{-2}{3}\right)$

DPP 2.6

Single Correct Answer Type

1. (b) a, b, c are roots of equation $\frac{1}{x}+\frac{1}{x-1}+\frac{1}{x-2}=1$

$\Rightarrow (x-1)(x-2)+x(x-2)+x(x-1)=x(x-1)(x-2)$

$\Rightarrow x^3-6x^2+8x-2=0$

2. (a) $1.2(\alpha+i\beta)(\alpha-i\beta)=p(0)=2\Rightarrow(\alpha+i\beta)(\alpha-i\beta)=1$

3. (c) Let $a+b=\lambda\Rightarrow b=\lambda-a$

So $(\lambda-a)^3-3(\lambda-a)^2+5(\lambda-a)+11=0$

$\lambda^3-a^3-3\lambda^2a+3\lambda a^2-3\lambda^2+6\lambda a-3a^2+5\lambda-5a+11=0$

$\Rightarrow a^3+(3-3\lambda)a^2+(3\lambda^2-6\lambda+5)a-(\lambda^3-3\lambda^2+5\lambda+11)=0$

Comparing it with the equation $a^3-3a^2+5a-17=0$, we get

$\lambda=2$

4. (c)

Let $f(x)=(x-x_1)(x-x_2)(x-x_3)(x-x_4)$

$\Rightarrow |f(i)|=\sqrt{1+x_1^2}\sqrt{1+x_2^2}\sqrt{1+x_3^2}\sqrt{1+x_4^2}=1$

$\Rightarrow x_1=x_2=x_3=x_4=0$

$\Rightarrow$ All four roots are zero.

$\Rightarrow f(x)=x^4\Rightarrow a+b+c+d=0$

5. (b) According to question

$f(x)-x=(x-1)(x-2)(x-3)(x-\alpha)$

$\Rightarrow f(-1)=24(1+\alpha)-1$

$f(0)=6\alpha$

$f(4)=6(4-\alpha)+4$

$f(5)=24(5-\alpha)+5$

$\Rightarrow \left[\frac{f(-1)+f(5)}{f(0)+f(4)}\right]=\left[\frac{148}{28}\right]=5.$

6. (b) $P(x)=x^6-x^5-x^3-x^2-x$ and $Q(x)=x^4-x^3-x^2-1$

Here $P(x)=Q(x)(x^2+1)+x^2-x+1$

$\therefore \Sigma P(\alpha)=\Sigma\alpha^2-\Sigma\alpha+4$

$=(\Sigma\alpha)^2-2\Sigma(\alpha\beta)-\Sigma\alpha+4$

$=1-2(-1)-(1)+4$

$=6$

7. (b) The solution of $-x^4+2x^2+x=mx+1$ is x_1, x_1 and x_2, x_2.

$\Rightarrow x^4-2x^2+(m-1)x+1=(x-x_1)^2(x-x_2)^2$

By comparing, we get

$x_1=1, x_2=-1, m=1\Rightarrow P(1,2)$ and $Q(-1,0)$

8. (d) $ab+bc+ca=\frac{(a+b+c)^2-(a^2+b^2+c^2)}{2}=183$

Hence a, b, c are the roots of the equation,

$t^3-24t^2+183t-440=0$

$\Rightarrow (t-5)(t-8)(t-11)=0$

Thus $\{a, b, c\}=\{5, 8, 11\}$

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9. (d) Let the 4 roots be $x-3y, x-y, x+y, x+3y$.

sum $= 4x$ and if $4x = 0$ then 4 roots are $-3y, -y, y, 3y$

and the product is $9y^4 = 225 \Rightarrow y^2 = 5$.

The product taken two at a time is $-10y^2$

$\Rightarrow \quad q = -50$

Multiple Correct Answers Type

10. (c, d)

Let $P = 3\alpha + 2$

$$\Rightarrow \quad \alpha = \frac{P-2}{3}$$

Since $9\alpha^3 - 7\alpha + 6 = 0$

$$\Rightarrow \quad \frac{9(P-2)^3}{27} - \frac{7}{3}(P-2) + 6 = 0$$

$$\Rightarrow \quad \frac{1}{3}(P^3 - 8 - 6P^2 + 12P) - \frac{7}{3}P + \frac{14}{3} + 6 = 0$$

$$\Rightarrow \quad P^3 - 6P^2 + 12P - 8 - 7P + 14 + 18 = 0$$

$$\Rightarrow \quad P^3 - 6P^2 + 5P + 24 = 0$$

So, the equation $x^3 - 6x^2 + 5x + 24 = 0$ has roots $3\alpha + 2, 3\beta + 2, 3\gamma + 2$.

11. (a, c)

Suppose that both r and $r + 1$ are solutions to the equation $x^3 + 3x^2 - 34x = m$

Then $r^3 + 3r^2 - 34r = m$, and also

$$(r+1)^3 + 3(r+1)^2 - 34(r+1) = m.$$

Subtracting the first of these equations from the second gives

$(3r^2 + 3r + 1) + 3(2r + 1) - 34 = 0$, or $3r^2 + 9r - 30 = 0$.

or $\quad r^2 + 3r - 10 = 0$

or $\quad (r+5)(r-2) = 0$

or $\quad r = -5$ or $r = 2$. If $r = -5$.

$\therefore \quad m = (-5)^3 + 3(-5)^2 - 34(-5)$

$= -125 + 75 + 170 = 120$

For $r = 2$, $m = -48$

12. (a, b, c, d)

Let $x = \alpha^2$, $\alpha = \sqrt{x}$, put this value in $x^3 + x + 1 = 0$

We get $x\sqrt{x} + \sqrt{x} + 1 = 0$

$$\sqrt{x}\,(x+1) = -1$$

$\Rightarrow \quad x(x+1)^2 = 1$

$\Rightarrow \quad x^3 + 2x^2 + x - 1 = 0$

$\Rightarrow \quad p(x) = x^3 + 2x^2 + x - 1$

$\Rightarrow \quad p(1) = 3$

Also $p(1)$ is odd, when n is odd or even.

Comprehension Type

13 (a), 14. (d)

The y-intercept is at $x = 0$, so we have $c = 2$, meaning that the product of the roots is -2.

We know that $-a$ is the sum of the roots.

The average of the roots is equal to the product, so the sum of the roots is -6, and $a = 6$.

Finally, $1 + a + b + c = 2$ as well, so we have $1 + 6 + b + 2 = -2$

$\Rightarrow \quad b = -11$

$\therefore \quad P(x) = x^3 + 6x^2 - 11x + 2$

$\therefore \quad P(1) = 1 + 6 - 11 + 2 = -2$

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CHAPTER 3

DPP 3.1

Single Correct Answer Type

1. (c) $S=\sum_{n=0}^{100}(i)^{n!}$

$=(i)^{0!}+(i)^{1!}+(i)^{2!}+\ldots$

$=i+i-1+i^6+i^{24}+(i)^{5!}+(i)^{6!}+\ldots+(i)^{100!}$

$=95+2i$

2. (c) $S=i+2i^2+3i^3+\ldots\ldots+ni^n$ (i)

$\therefore\quad iS=i^2+2i^3+3i^4+\ldots\ldots+ni^{n+1}$ (ii)

Subtracting , we get

$S(1-i)=i+i^2+i^3+\ldots\ldots\ldots+i^n-ni^{n+1}$

$=\dfrac{i(1-i^n)}{1-i}-ni^{n+1}$

Now put the values of n and verify.

3. (b) $z_1=0, z_{n+1}=z_n^2+i$

Put $n=1\Rightarrow z_2=z_1^2+i=i$

Put $n=2\Rightarrow z_3=z_2^2+i=i^2+i=i-1$

Put $n=3\Rightarrow z_4=z_3^2+i=(i-1)^2+i=-i$

Put $n=4\Rightarrow z_5=z_4^2+i=(-i)^2+i=i-1$

Put $n=5\Rightarrow z_6=z_5^2+i=(i-1)^2+i=-i$

Thus $z_{111}=-1+i$

4. (b) $\dfrac{(-\sqrt{3}+3i)(1-i)}{(3+\sqrt{3}i)(i)(\sqrt{3}+\sqrt{3}i)}$

$=\dfrac{\sqrt{3}(-1+\sqrt{3}i)(1-i)}{3(\sqrt{3}+i)(i)(1+i)}$

$=\dfrac{(-1+\sqrt{3}i)(1-i)}{\sqrt{3}(-1+\sqrt{3}i)(1+i)}$

$=\dfrac{(1-i)^2}{2\sqrt{3}}=\dfrac{-2i}{2\sqrt{3}}$, which lies on imaginary axis

5. (c) Let α be the root. Then $(a\alpha^2+2b\alpha+c)+i(d\alpha^2+2e\alpha+f)=0$

$\Rightarrow\quad a\alpha^2+2b\alpha+c=0\Rightarrow\alpha=-\sqrt{\dfrac{c}{a}}$ (as a, b, c are in G.P.)

$\therefore$ From $d\alpha^2+2e\alpha+f=0$

$d.\dfrac{c}{a}-2e\dfrac{\sqrt{c}}{\sqrt{a}}+f=0$

$\Rightarrow\quad\dfrac{d}{a}-2.\dfrac{e}{b}+\dfrac{f}{c}=0$

6. (c) Suppose x is a real root.

Then $x^3+ix-1=0$

$\Rightarrow\quad x^3-1=0$ and $x=0$

$\Rightarrow$ There is no real number satisfying these two equations.

7. (b) Let us consider α as the real and $i\beta$ as the imaginary root. Then

$\alpha+i\beta=-a$

$\Rightarrow\quad\alpha-i\beta=-\bar{a}$

$\Rightarrow\quad 2\alpha=-(a+\bar{a})$ and $2i\beta=-(a-\bar{a})$

$\Rightarrow\quad 4i\alpha\beta=a^2-\bar{a}^2$

$\Rightarrow\quad a^2-\bar{a}^2=4b$

8. (c) Let $Z=a+ib, b\neq 0$ where Im $Z=b$

$Z^5=(a+ib)^5$

$=a^5+{}^5C_1\,a^4\,bi+{}^5C_2\,a^3\,b^2i^2+{}^5C_3\,a^2b^3i^3+{}^5C_4\,ab^4i^4+i^5b^5$

Im $Z^5=5a^4b-10a^2b^3+b^5$

$y=\dfrac{\text{Im}\,Z^5}{\text{Im}^5\,Z}=5\left(\dfrac{a}{b}\right)^4-10\left(\dfrac{a}{b}\right)^2+1$

Let $\left(\dfrac{a}{b}\right)^2=x$ (say), $x\in R^+$

$y=5x^2-10x+1$

$=5[x^2-2x]+1$

$=5[(x-1)^2]-4$

Hence, $y_{\min}=-4$

9. (a) $z_1\left(\dfrac{\bar{z}_2z_3-z_2\bar{z}_3}{2i}\right)+z_2\left(\dfrac{\bar{z}_3z_1-z_3\bar{z}_1}{2i}\right)+z_3\left(\dfrac{\bar{z}_1z_2-z_1\bar{z}_2}{2i}\right)=\dfrac{1}{2i}\times 0=0$

Multiple Correct Answers Type

10. (b, c)

(i) $xy>0, 3x^2-xy-2y^2=-7$ or $(3x+2y)(x-y)=-7$

x and y being integers, we can take

$3x+2y=7$ and $x-y=-1$

$x=1, y=2$

If x and y are charged to $-x, -y$, equation remains same

$x=-1, y=-2$ is also a solution pair.

$3x+2y=-1$ and $x+y=7$ do not give integral solutions.

(ii) $xy=0$ will not give any integral solution.

(iii) $xy<0\qquad 3x^2+xy-2y^2=-7$

$(3x-2y)(x+y)=-7$

$3x-2y=-7$ and $x+y=1$ leads to $x=-1\ y=2$

$3x-2y=7$ and $x+y=-1$ lead to $x=1$ and $y=-2$.

Required complex numbers are $1+2i, 1-2i, -1+2i, -1-2i$ which form two conjugate pairs.

11. (a, b, c)

Let $z_1=x_1, z_2, z_3=x_2\pm iy_2$

$\Rightarrow\quad z_1+z_2+z_3=-\dfrac{b}{a}$

$\Rightarrow\quad x_1+2x_2=-\dfrac{b}{a}<0\Rightarrow ab>0$

Also, $z_1z_2z_3=x_1[x_2^2+y_2^2]=-\dfrac{d}{a}<0\Rightarrow ad>0$

Also $z_1z_2+z_2z_3+z_1z_3=\dfrac{c}{a}$

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$\Rightarrow x_1(x_2+iy_2)+x_1(x_2-iy_2)+x_2^2+y_2^2=2x_1x_2+x_2^2+y_2^2>0$

$\Rightarrow \quad \frac{c}{a}>0$

$\Rightarrow \quad \frac{b}{a}\frac{c}{a}>0 \Rightarrow bc>0$

12. (a, b)

Let r be the common ratio of G.P. a, b, c we have

$$\therefore \quad z=\frac{\frac{a}{b}+i}{\frac{c}{b}-i}=\frac{\frac{1}{r}+i}{r-i}=\frac{i}{r}$$

$$z=\frac{ib}{c} \text{ or } \frac{ia}{b}$$

DPP 3.2

Single Correct Answer Type

1. (c) $\frac{1+2i}{1-(1-i)^2}=\frac{1+2i}{1-1-i^2+2i}=\frac{1+2i}{1+2i}=1+i.0$

$\therefore$ Modulus = 1

Amplitude = $\tan^{-1}\left|\frac{0}{1}\right|=0$

2. (b) $(z-a)(\bar{z}-b)=z\bar{z}-a\bar{z}-bz+ab \qquad (\because z=x+iy)$

Argument of $(z-a)(\bar{z}-b)=\left(\frac{y(a-b)}{x^2+y^2-ax-bx+ab}\right)$

Also, $\frac{(\sqrt{3}+i)(1+\sqrt{3}i)}{1+i}=\frac{\sqrt{3}-\sqrt{3}+i(1+3)}{1+i}=2+2i$

Argument of $2+2i=\tan^{-1}(2/2)=\pi/4$

$\Rightarrow \quad x^2+y^2-(a+b)x-(a-b)y+ab=0$

Thus, the locus of z is a circle

Given centre is $\frac{3+i}{2}$

$\Rightarrow \quad \frac{a+b}{2}=\frac{3}{2}$ and $\frac{a-b}{2}=\frac{1}{2}$

$\Rightarrow \quad a=2, b=1$

3. (c) Let $z=r(\cos\theta+i\sin\theta)$

Then the given equation is $r^2+\frac{4}{r^2}-2.2\cos 2\theta-16=0$

$\Rightarrow \quad r^4-4r^2(\cos 2\theta+4)+4=0$

$\Rightarrow \quad r^2=2(\cos 2\theta+4)+2\sqrt{(\cos 2\theta+4)^2-1}$

The maximum value is obtained when $\cos 2\theta=1$

$\therefore$ The maximum value of $r^2=10+2\sqrt{24}$

$=(2+\sqrt{6})^2$

$\Rightarrow$ The maximum value of $r=2+\sqrt{6}$

4. (a) $a=e^{i\alpha}, b=e^{i\beta}, c=e^{i\gamma}$

Clearly $a+b+c=0$

$\Rightarrow \quad a+b+c=0=\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$

$\Rightarrow \quad a^3+b^3+c^3=3abc$

$\Rightarrow \quad \sin 3\alpha+\sin 3\beta+\sin 3\gamma=\sin(\alpha+\beta+\gamma)$

5. (d) Let $z=x+iy$

Then $|z-3-4i|^2+|z+2-7i|^2+|z-5+2i|^2$

$=3(x^2+y^2-4x-6y)+107$

$=3[\{(x-2)^2+(y-3)^2]+68$

$\therefore$ The least value occurs when $x=2$ and $y=3$.

$\therefore \quad z=2+3i$

6. (d) $x^4-2x^2+4=0$

$\Rightarrow \quad x^2=1\pm\sqrt{3}i$

$=2e^{\pm\frac{\pi}{3}}$

$\Rightarrow \quad x=\pm\sqrt{2}\,e^{\pm i\frac{\pi}{6}}$

These points lie on a circle of radius $\sqrt{2}$, as the diagonals intersect at an angle $\pi/3$,

$\therefore$ These are the vertices of an inscribed rectangle.

7. (b) Let $z_1=x_1+iy_1$ and $z_2=x_2+iy_2$

Given $\text{Re}(z_1)=|z_1-2|$, $\text{Re}(z_2)=|z_2-2|$

$\therefore \quad y_1^2-4x_1+4=0$ and $y_2^2-4x_2+4=0$

So that $\frac{y_1-y_2}{x_1-x_2}=\frac{4}{y_1+y_2}$ (i)

Given $\arg(z_1-z_2)=\pi/3$

$\Rightarrow \quad \frac{y_1-y_2}{x_1-x_2}=\sqrt{3}$ (ii)

From (i) and (ii) $\Rightarrow \quad y_1+y_2=\frac{4}{\sqrt{3}}$

8. (c) $(1+z+z^2+z^3+z^4)+4z^4(1+z+z^2+z^3+z^4)+5z^9.$

$=\frac{1-z^5}{1-z}+4z^4\left(\frac{1-z^5}{1-z}\right)+5z^4$

$=5z^4 \qquad (\because z^5=1)$

9. (d) $z=(3a-7b)+i(3b+7a)$

For purely imaginary, $3a=7b$

$\Rightarrow \quad a=7$ or $b=3$ (for least value of $|z|$)

$\Rightarrow \quad |z|=|3+7i|=\sqrt{58}$

DPP 3.3

Single Correct Answer Type

1. (a) $|z+16|^2=16|z+1|^2$

$\Rightarrow \quad (z+16)(\bar{z}+16)=16(z+1)(\bar{z}+1)$

$\Rightarrow \quad z\bar{z}+16z+16\bar{z}+256=16z\bar{z}+16z+16\bar{z}+16$

$\Rightarrow \quad z\bar{z}=16 \Rightarrow |z|^2=16 \Rightarrow |z|=4$

2. (d) $z'=\frac{1+z^2}{z}=\frac{z\bar{z}+z}{z}=z+\bar{z}$ which is purely real.

$\Rightarrow \quad \text{Im}(z')=0.$

3. (c) $a\bar{a}=b\bar{b}=c\bar{c}=1$

$\therefore \quad \bar{a}=\frac{1}{a}$ etc.

Now, $|abc|=|a+b+c|$

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$$= |\bar{a} + \bar{b} + \bar{c}|$$
$$= \left|\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right|$$
$$= \left|\frac{\Sigma ab}{abc}\right|$$

$\therefore$ $|\Sigma ab| = |abc|\,|abc| = (|a|\,|b|\,|c|)^2 = 1$

4. (b) Let $y = |z_1 - z_2|^2 + |z_2 - z_3|^2 + |z_3 - z_1|^2$

$= (z_1 - z_2)(\bar{z}_1 - \bar{z}_2) + (z_2 - z_3)(\bar{z}_2 - \bar{z}_3) + (z_3 - z_1)(\bar{z}_3 - \bar{z}_1)$

$= 6 - (z_1\bar{z}_2 + z_2\bar{z}_1 + z_2\bar{z}_3 + \bar{z}_2 z_3 + z_3\bar{z}_1 + z_1\bar{z}_3)$

Now we know $|z_1 + z_2 + z_3|^2 \geq 0$ (i)

$\Rightarrow$ $3 + (z_1\bar{z}_2 + z_2\bar{z}_1 + z_1\bar{z}_3 + z_3\bar{z}_1 + z_2\bar{z}_3 + \bar{z}_2 z_3) \geq 0$ (ii)

From (i) and (ii), $y \leq 9$.

5. (c) Let $z = a + ib$

$\Rightarrow$ $\bar{z} = a - ib$

Hence, we have $z^{2008} = \bar{z}$

$\therefore$ $|z|^{2008} = |\bar{z}| = |z|$

$\therefore$ $|z|\,[|z|^{2007} - 1] = 0$

$\therefore$ $|z| = 0$ or $|z| = 1$;

If $|z| = 0 \Rightarrow z = 0$

If $|z| = 1 z^{2009} = z\bar{z} = |z|^2 = 1$

$\Rightarrow$ 2009 values of z

$\Rightarrow$ Total = 2010

6. (b) $|2z - 3i|^2 < |3z - 2i|^2$

$\therefore$ $4x^2 + (2y - 3)^2 < 9x^2 + (3y - 2)^2$

$\therefore$ $x^2 + y^2 > 1$

$\therefore$ $|z|^2 > 1$

7. (b) $z\omega = |z|^2$

$\Rightarrow$ $z\omega = z\bar{z}$

$\Rightarrow$ $\omega = \bar{z}$

$\therefore$ $|z - \bar{z}| + |z + \bar{z}| = 4$

$\therefore$ $|x| + |y| = 2$

which is a square $\therefore$ Area = 8 sq. units

8. (d) $a\alpha^2 + b\alpha + 1 = 0$ (i)

$\Rightarrow$ $\bar{a}\bar{\alpha}^2 + \bar{b}\bar{\alpha} + 1 = 0$

$\Rightarrow$ $\alpha^2 + \bar{b}\alpha + \bar{a} = 0$ (as $|\alpha| = \alpha\bar{\alpha} = 1$) (ii)

From (i) and (ii),

$$\frac{\alpha^2}{\bar{a}b - \bar{b}} = \frac{\alpha}{1 - |a|^2} = \frac{1}{a\bar{b} - b} \Rightarrow |a\bar{b} - \bar{b}| = 1 - |a|^2 = 1 - \frac{1}{4} = \frac{3}{4}$$

9. (a) $z^2 + pz + q = 0$

$z_1 + z_2 = p$ and $z_1 z_2 = q$

Given $|p| + |q| < 1$

$\therefore$ $|z_1 + z_2| + |z_1 z_2| < 1$ (i)

but $||z_1| - |z_2|| \leq |z_1 + z_2|$ (ii)

Form (i) and (ii)

$||z_1| - |z_2|| \leq |z_1 + z_2| < 1 - |z_1||z_2|$ {$1 - |z_1||z_2|$ must be positive}

If $|z_1| - |z_2| - 1 + |z_1||z_2| < 0$

$\therefore$ $(|z_1| - 1) + |z_2|(|z_1| - 1) < 0$

$(|z_1| - 1)(|z_2| + 1) < 0$ (iii)

Also $|z_2| - |z_1| - 1 + |z_1||z_2| < 0$

$(|z_2| - 1)(|z_1| + 1) < 0$ (iv)

From (iii) and (iv)

$|z_1| < 1$ and $|z_2| < 1$

10. (a) $z^3 = -\omega^5 \Rightarrow |z|^3 = |\omega|^5 \Rightarrow |z|^6 = |\omega|^{10}$ (i)

and $z^2 = \frac{1}{\bar{\omega}^4} \Rightarrow |z|^2 = \frac{1}{|\omega|^4} \Rightarrow |z|^6 = \frac{1}{|\omega|^{12}}$ (ii)

From (i) and (ii), $|\omega| = 1$ and $|z| = 1 \Rightarrow z\bar{z} = \omega\bar{\omega} = 1$

Again $z^6 = \omega^{10}$ (iii)

and $z^6 \cdot \bar{\omega}^{12} = 1$

$$z^6 = \frac{1}{\bar{\omega}^{12}} = \omega^{10} \text{ (using (iii))}$$

$\Rightarrow$ $(\omega\,\bar{\omega})^{10}(\bar{\omega})^2 = 1$

$\Rightarrow$ $(\bar{\omega})^2 = 1$

$\Rightarrow$ $\bar{\omega} = 1$ or $-1 \Rightarrow \omega = 1$ or -1

If $\omega = 1$, then $z^3 + 1 = 0$ and $z^2 = 1 \Rightarrow z = -1$

If $\omega = -1$, then $z^3 - 1 = 0$ and $z^2 = 1 \Rightarrow z = 1$

Hence, $z = 1$ and $\omega = -1$ or $z = -1$ and $\omega = 1$

11. (d) Given, $|z - |z + 1||^2 = |z + |z - 1||^2$

$\therefore$ $(z - |z + 1|)(\bar{z} - |z + 1|) = (z + |z - 1|)(\bar{z} + |z - 1|)$

$z\bar{z} - z|z + 1| - \bar{z}|z + 1| + |z + 1|^2$

$= z\bar{z} + z|z - 1| + \bar{z}|z - 1| + |z - 1|^2$

$|z + 1|^2 - |z - 1|^2 = (z + \bar{z})[|z - 1| + |z + 1|]$

$(z + 1)(\bar{z} + 1) - (z - 1)(\bar{z} - 1) = (z + \bar{z})[|z - 1| + |z + 1|]$

$(z\bar{z} + z + \bar{z} + 1) - (z\bar{z} - z - \bar{z} + 1) = (z + \bar{z})[|z - 1| + |z + 1|]$

$2(z + \bar{z}) = (z + \bar{z})[|z + 1| + |z - 1|]$

$(z + \bar{z})[|z + 1| + |z - 1| - 2] = 0$

$\Rightarrow$ Either $z + \bar{z} = 0$ $\Rightarrow$ z is purely imaginary.

$\Rightarrow$ z lies on y axis $\Rightarrow$ $x = 0$

or $|z + 1| + |z - 1| = 2$

$\Rightarrow$ z lies on the line segment joining $(-1, 0)$ and $(1, 0)$.

12. (d) $\frac{z_1 - z_2}{z_1 + z_2} = \cos\alpha + i\sin\alpha$

$\Rightarrow$ $\frac{2z_1}{-2z_2} = \frac{\cos\alpha + i\sin\alpha + 1}{\cos\alpha - 1 + i\sin\alpha}$

$\frac{2\cos^2\alpha/2 + 2i\sin\alpha/2\cos\alpha/2}{2i\sin\alpha/2\cos\alpha/2 - 2\sin^2\alpha/2}$

$\frac{2\cos\alpha/2[\cos\alpha/2 + i\sin\alpha/2]}{2i\sin\alpha/2[\cos\alpha/2 + i\sin\alpha/2]}$

$\Rightarrow$ $\frac{z_1}{z_2} = i\cot\frac{\alpha}{2}$

$\Rightarrow$ Given $\frac{z_1}{z_2} = \frac{K}{1}$

$\therefore$ $\tan\alpha/2 = -1/K$

$\tan\alpha = \frac{2\tan\alpha/2}{1 - \tan^2\alpha/2} \Rightarrow \frac{-2/K}{1 - 1/K^2} \Rightarrow \frac{-2K}{K^2 - 1}$

$\alpha = \tan^{-1}\left(\frac{2K}{1 - K^2}\right) \Rightarrow 2\tan^{-1}(K)$

Multiple Correct Answers Type

13. (b, d)

$a + \bar{c} \neq 0$

Now, $(a + \bar{c})w_1^2 + (b + \bar{b})w_1 + (\bar{a} + c) = 0$

$\Rightarrow$ $(\bar{a} + c)\bar{w}_1^2 + (b + \bar{b})\bar{w}_1 + (a + \bar{c}) = 0$

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Since, $\overline{w}_1 \neq 0$, we get $(a+\overline{c})\frac{1}{\overline{w}_1^2}+(b+\overline{b})\frac{1}{\overline{w}_1}+(\overline{a}+c)=0$

Hence, $\frac{1}{\overline{w}_1}$ is also a root, but $w_2 \neq \frac{1}{\overline{w}_1}$, so $w_1 = \frac{1}{\overline{w}_1}$

$\Rightarrow \quad |w_1| = 1$

Similarly $|w_2| = 1$.

14. (a, c)

$|Z^2 - 9| + |Z^2| = 41$

$\Rightarrow \quad |Z+3| + |Z-3| = 10$

$\Rightarrow \quad 10 = |Z+3| + |Z-3| \geq |Z+3+Z-3|$

$\Rightarrow \quad |Z| \leq 5$

15. (a, b)

$$|z_1+z_2| = \left|-\frac{b}{a}\right| = 1$$

$$|z_1 z_2| = \left|\frac{c}{a}\right| = 1$$

$\therefore \quad |z_2| = 1$

$|z_1+z_2|^2 = 1$

$\therefore \quad 2 + z_1\overline{z}_2 + z_2\overline{z}_1 = 1$

Now $z_2 = z_1 e^{i\theta}$

$\therefore \quad |z_1 + z_1 e^{i\theta}| = |z_1||1+e^{i\theta}| = 1$

$$\therefore \quad 2\cos\frac{\theta}{2} = 1$$

$$\therefore \quad \theta = \frac{2\pi}{3}$$

Now, $\frac{(z_1+z_2)^2}{z_1 z_2} = 1$

$$\Rightarrow \quad \frac{b^2}{a^2} = \frac{c}{a}$$

$\Rightarrow \quad b^2 = ac$

$PQ = |z_1 - z_2| = \sqrt{3}$

DPP 3.4

Single Correct Answer Type

1. (a) $z + \frac{1}{z} = 2\cos 6^\circ$

$\therefore \quad z = \cos 6^\circ \pm i\sin 6^\circ$

$$\Rightarrow \quad z^{1000} + \frac{1}{z^{1000}} + 1 = 2\cos 6000^\circ + 1$$

$$= 2 \times \left(-\frac{1}{2}\right) + 1 = 0$$

2. (c) $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2) + 2m\pi,\ m \in I$

$\therefore$ Principal argument $= \alpha + \beta - 2\pi$

3. (b) $\arg(z_1 z_2 z_3 \ldots z_n) = \pi$

$\Rightarrow \quad \arg(z_1) + \arg(z_2) + \ldots + \arg(z_n) = \pi \pm 2m\pi,\ m \in I$

$$\Rightarrow \quad \frac{\pi}{n}[3+5+7+\ldots+(2n+1)] = \pi \pm 2m\pi$$

$$\Rightarrow \quad \frac{\pi}{n}\left[\frac{n}{2}[6+2(n-1)]\right] = \pi \pm 2m\pi$$

$\Rightarrow \quad 3 + n - 1 = 1 \pm 2m$

$\Rightarrow \quad n = -1 \pm 2m.$

4. (d) $(z+w)^2 = zw$

$\Rightarrow \quad z^2 + zw + w^2 = 0$

Let $\frac{z}{w} = t \Rightarrow \frac{z}{w} = \frac{-1 \pm \sqrt{3}i}{2}$

$\arg z - \arg w = \frac{2\pi}{3}$ or $\arg z - \arg w = -\frac{2\pi}{3}$

5. (c) $\arg(z) - \arg(z+1) - \arg(z-1)$

$$= \arg\left(\frac{z}{z^2-1}\right)$$

$$= \arg\left(\frac{z}{z^2 - z\overline{z}}\right)$$

$$= \arg\left(\frac{1}{z-\overline{z}}\right)$$

= arg (purely imaginary no.)

6. (d) We have

$$\arg(z^{3/8}) = \frac{1}{2}\arg(z^2 + \overline{z}z^{1/2})$$

$\Rightarrow \quad 2\arg(z^{3/8}) = \arg(z^2 + \overline{z}z^{1/2})$

$\Rightarrow \quad \arg(z^{3/4}) - \arg(z^2 + \overline{z}z^{1/2}) = 0 \qquad [\because 2\arg(z) = \arg(z^2)]$

$$\Rightarrow \quad \arg\left(\frac{z^2 + \overline{z}z^{1/2}}{z^{3/4}}\right) = 0$$

$$\Rightarrow \quad \mathrm{Im}\left(z^{5/4} + \frac{\overline{z}}{z^{1/4}}\right) = 0$$

$$\Rightarrow \quad z^{5/4} + \frac{\overline{z}}{z^{1/4}} = \overline{\left(z^{5/4} + \frac{\overline{z}}{z^{1/4}}\right)}$$

$$\Rightarrow \quad z^{5/4} + \frac{\overline{z}}{z^{1/4}} = (\overline{z})^{5/4} + \frac{z}{(\overline{z})^{1/4}}$$

$$\Rightarrow \quad z^{5/4} + \frac{\overline{z}(\overline{z})^{1/4}}{|z|^{1/2}} = (\overline{z})^{5/4} + \frac{z z^{1/4}}{|z|^{1/2}}$$

$$\Rightarrow \quad z^{5/4} - (\overline{z})^{5/4} = \frac{z^{5/4} - (\overline{z})^{5/4}}{|z|^{1/2}}$$

$$\Rightarrow \quad \{z^{5/4} - (\overline{z})^{5/4}\}\left(1 - \frac{1}{|z|^{1/2}}\right) = 0$$

$\therefore \quad z = \overline{z}$ or $|z| = 1$.

7. (b)

$$z = \frac{2z_1}{3z_2} + \frac{3z_2}{2z_1}$$

$$= \frac{2}{3}\frac{|z_1|}{|z_2|}e^{i(\theta_1-\theta_2)} + \frac{3}{2}\frac{|z_2|}{|z_1|}e^{i(\theta_2-\theta_1)}$$

$$= e^{i(\theta_1-\theta_2)} + e^{i(\theta_2-\theta_1)} = 2\cos(\theta_1-\theta_2) = 2\cos\alpha$$

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Multiple Correct Answers Type

8. (a, b, c, d)

$Z_1 = e^{i\theta_1}, Z_2 = e^{i\theta_2}, \text{Re}(Z_1 Z_2) = 0 \Rightarrow \theta_1 + \theta_2 = -\pi/2$, (as z_1, z_2 lie in fourth quadrant)

$Z_3 = e^{-i\theta_1}, Z_4 = -ie^{i\theta_1}, Z_5 = \cos\theta_1(1-i), Z_6 = \sin\theta_1(-1+i)$

9. (a, c)

Let $z - 3 = re^{i\phi}$

$$\therefore \quad \frac{z-3}{e^{i\theta}} + \frac{e^{i\theta}}{z-3} = re^{i(\phi-\theta)} + \frac{1}{r}e^{i(\theta-\phi)}$$

Imaginary part of the above $= r\sin(\phi-\theta) - \frac{1}{r}\sin(\phi-\theta)$

Given that $r\sin(\phi-\theta) - \frac{1}{r}\sin(\phi-\theta) = 0$

$$\Rightarrow \quad r - \frac{1}{r} = 0 \text{ or } \sin(\phi-\theta) = 0$$

$$\Rightarrow \quad r - \frac{1}{r} = 0 \Rightarrow r = 1$$

$\Rightarrow |z-3| = 1$

$\Rightarrow$ z lies on a circle with unit radius.

$\sin(\phi-\theta) = 0 \Rightarrow \phi = \theta$

$\therefore \quad z - 3 = re^{i\theta}$

$z - 3 = r\cos\theta, y = r\sin\theta$

$\Rightarrow \quad x - 3 = r\cos\theta, y = r\sin\theta$

$$\frac{x-3}{\cos\theta} = \frac{y}{\sin\theta} = r$$

This represents a straight line through (3, 0).

DPP 3.5

Single Correct Answer Type

1. (c) As $\left|\frac{a\alpha + b\beta + c\gamma}{a\beta + b\gamma + c\alpha}\right| = 1$

$\Rightarrow$ When α, β, γ are different, then number of triplet (α, β, γ) = permutation of 1, ω and $\omega^2 = 6$ and when $\alpha = \beta = \gamma$, number of triplets = 3.

2. (b) $\left(2(\sqrt[3]{3})\left(\frac{1+i\sqrt{3}}{2}\right)\right)^3 = 8 \times 3(-\omega)^3 = -24$

3. (b) $(a\omega^2 + b\omega)(a\omega + b\omega^2) = a^2 - ab + b^2$

$$= \frac{(a+b)(a^2 - ab + b^2)}{a+b}$$

$$= \frac{a^3 + b^3}{a+b} = \frac{105}{21} = 5$$

4. (d) $z = \frac{1}{2}(\sqrt{3} - i) = iw^2$, where w is imaginary cube root of unity.

$\Rightarrow \quad (z^{101} + i^{109})^{106}$

$= ((i\omega^2)^{101} + i^{109})^{106}$

$= (i\omega^2 + i)^{106}$

$= (-i\omega)^{106}$

$= -\omega^2$

Also $z^{m+1} = (\omega^2 i)^{m+1}$

$\Rightarrow \quad m = 9$ satisfies.

5. (c) We have $||z_1| - |z_2|| \le |z_1 - z_2|$ and equality holds only when $\arg z_1 = \arg z_2$

$\Rightarrow \quad ||z - w| - |z - w^2|| \le |w^2 - w| \le \sqrt{3}$ and equality can hold only when $|z| = 2$ and not when $|z| = \frac{1}{2}$.

6. (b) Roots are $\alpha = 2\omega^2, \beta = 3 + 4\omega, \gamma = 3 + 4\omega^2, \delta = 5 - \omega - \omega^2$

$\delta = 5 - (\omega + \omega^2) = 5 - (-1) = 6$

If $\alpha = 2\omega^2$ is a root, then 2ω has to be a root too.

Minimum 5 roots, hence minimum degree $\rightarrow 5$.

7. (c) $z^2 = \bar{z} \cdot 2^{1-|z|}$

$\therefore \quad z^3 = |z|^2 2^{1-|z|}$ (i)

$\Rightarrow \quad |z| = 2^{1-|z|}$ (by taking modulus both sides)

$\Rightarrow \quad |z| = 1$

$\Rightarrow \quad z^3 = 1$

$\Rightarrow \quad z = 1, \omega, \omega^2$

But 1 is not imaginary

Hence $z = w$ or w^2

8. (b) $z^4 = 2(1 - \sqrt{3}i) = 4\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)$

$$= 4\left[\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\right]$$

$$z = \sqrt{2}\left[\cos\frac{2m\pi - (\pi/3)}{4} + i\sin\frac{2m\pi - (\pi/3)}{4}\right]$$

For $m = 1, z = \sqrt{2}\left[\cos\left(\frac{5\pi}{12}\right) + i\sin\left(\frac{5\pi}{12}\right)\right]$.

9. (b) Let $\alpha (|\alpha| = 1)$ is a root of unity that is also a root of the equation $z^2 + az + b = 0$

$\therefore \quad \alpha + \bar{\alpha} = -a$

$\therefore \quad |a| = |\alpha + \bar{\alpha}| \le |\alpha| + |\bar{\alpha}| = 2$

and $b = \alpha\bar{\alpha} = 1$

Hence, we must check those equations for which $-2 \le a \le 2$ and $b = 1$.

i.e. $z^2 + 2z + 1 = 0$; $z^2 + z + 1 = 0$; $z^2 + 1 = 0$

$z^2 - 2z + 1 = 0$; $z^2 - z + 1 = 0$

Hence, roots are $\pm 1, \pm i; \frac{-1 \pm \sqrt{-3}}{2}, \frac{1 \pm \sqrt{-3}}{2}$.

10. (c) Given equation is $Z^6 + Z^3 + 1 = 0$

Let $Z^3 = t$

Hence, equation becomes

$t^2 + t + 1 = 0$

$\Rightarrow \quad t = \omega$ or ω^2

$$\therefore \quad Z^3 = \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3} = e^{\left(2n\pi + \frac{2\pi}{3}\right)i}$$

$$\therefore \quad Z = e^{\frac{\left(2n\pi + \frac{2\pi}{3}\right)}{3}i}$$

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Putting $n = 1$, we get $\theta = \frac{8\pi}{9}$ $(= 160°) \in (90°, 180°)$

11. (b) Let $A = x + iy$;

$A^n = (A + 1)^n = 1$

$\Rightarrow |A| = 1$ (i)

$\Rightarrow x^2 + y^2 = 1$

And $|A + 1| = 1$ (ii)

$\Rightarrow (x + 1)^2 + y^2 = 1$

$\Rightarrow x = -\frac{1}{2}$ and $y = \pm\frac{\sqrt{3}}{2}$

$\Rightarrow A = \omega$ or ω^2

$\Rightarrow (\omega)^n = (1 + \omega)^n = (-\omega^2)^n$

$\Rightarrow n$ must be even and divisible by 3.

$\Rightarrow$ Least value of n is 6.

12. (a)

Let vertices be $1, \alpha, \alpha^2, \ldots, \alpha^{n-1}$.

Given $1 + \alpha + \alpha^2 + \ldots + \alpha^{n-1} = 0 \Rightarrow \alpha^n - 1 = 0$

$\Rightarrow$ $z_1, z_2, z_3, \ldots, z_n$ are roots of $\alpha^n = 1$

which form regular polygon. So distance is zero.

Multiple Correct Answers Type

13. (a, b, c)

We have $\alpha = \cos\frac{2\pi}{5} + i\sin\frac{2\pi}{5}$

and $1 + \alpha + \alpha^2 + \alpha^3 + \alpha^4 = 0$

$\therefore \quad |1 + \alpha + \alpha^2 + \alpha^3| = |-\alpha^4| = |\alpha|^4 = 1$

Also, $|1 + \alpha + \alpha^2| = |-\alpha^3(1 + \alpha)| = |1 + \alpha|$

$$= \left|1 + \cos\frac{2\pi}{5} + i\sin\frac{2\pi}{5}\right|$$

$$= \left|2\cos\frac{\pi}{5}\left(\cos\frac{\pi}{5} + i\sin\frac{\pi}{5}\right)\right|$$

$$= 2\cos\frac{\pi}{5}$$

Again, from (i), $|1 + \alpha| = 2\cos\frac{\pi}{5}$

14. (a, b)

Given equation $z^6 = (z + 1)^6$

$\Rightarrow |z^6| = |(z + 1)^6|$

$\Rightarrow |z| = |z + 1|$

$\Rightarrow$ Roots are collinear.

$\Rightarrow \arg\left(\frac{z_1 - z_3}{z_2 - z_3}\right) = 0$ or π

DPP 3.6

Single Correct Answer Type

1. (b) Let $w = 5(z - i) - 6$

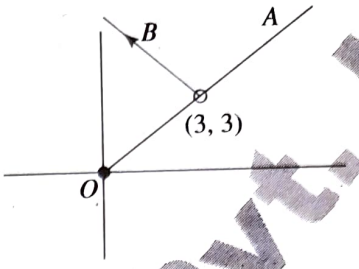
$\Rightarrow |w + 1| = 5|z - 1 - i| = 5$

2. (d) $A = \left\{z, \arg(z) = \frac{\pi}{4}\right\}$

z lies on the ray from origin (excluding origin) at an angle $\pi/4$ with positive real axis.

$$B = \left\{z, \arg(z - 3 - 3i) = \frac{2\pi}{3}\right\}$$

z lies on the ray from $3 + 3i$ (excluding $3 + 3i$) at an angle $2\pi/3$ with positive real axis.



We can observe that $3 + 3i \in A$ but $\notin B$

$\therefore \quad n(A \cap B) = 0$

3. (b) If z_1, z_2, z_3 are collinear and $az_1 + bz_2 + cz_3 = 0$. Then $a + b + c = 0$.

Hence, $1 + |\sin\theta| + |\cos\theta| - 1 - \sqrt{2} = 0$

$\Rightarrow |\sin\theta| + |\cos\theta| = \sqrt{2}$

$\Rightarrow \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

4. (d) $z^2 + az + a^2 = 0$

$\Rightarrow z = a\omega, a\omega^2$ (where 'ω' is non-real root of cube unity)

$\Rightarrow$ Locus of z is a pair of straight lines

and $\arg(z) = \arg(a) + \arg(\omega)$ or $\arg(a) + \arg(\omega^2)$

$\Rightarrow \arg(z) = \pm\frac{2\pi}{3}$

Also, $|z| = |a||\omega|$ or $|a||\omega^2| \Rightarrow |z| = |a|$.

5. (c) Given $\frac{1 + \bar{z}}{z}$ is real $\Rightarrow \frac{1 + \bar{z}}{z} = \frac{1 + z}{\bar{z}}$

$\Rightarrow \bar{z} + \bar{z}^2 = z + z^2 \Rightarrow (\bar{z} - z) + (\bar{z} - z)(\bar{z} + z) = 0$

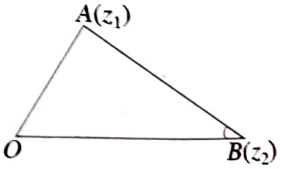
$\Rightarrow (\bar{z} - z)(1 + \bar{z} + z) = 0,$

So either $\bar{z} = z (z \neq 0)$ or $z + \bar{z} + 1 = 0$

$\Rightarrow y = 0$ or $x = \frac{-1}{2}$ but excluding origin.

6. (c) $\frac{z_1}{z_2} + \frac{\bar{z}_1}{\bar{z}_2} = 2$

$\Rightarrow \text{Re}\left(\frac{z_1}{z_2}\right) = 1$

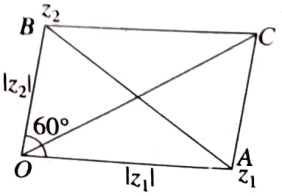


Let $\frac{z_1}{z_2} = x + iy \Rightarrow x = 1$

Now, $\arg\left(\frac{z_1 - z_2}{-z_2}\right) = \arg\left(1 - \frac{z_1}{z_2}\right) = \arg(-iy) = \pm\frac{\pi}{2}$

$\angle ABO = \frac{\pi}{2}$.

7. (c)



$OC = |z_1 + z_2|$ and $AB = |z_1 - z_2|$

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Using cosine rule

$$|z_1+z_2|=\sqrt{|z_1|^2+|z_2|^2-2|z_1||z_2|\cos 120^\circ}$$
$$=\sqrt{4+9+2\cdot 3}=\sqrt{19}$$

and $$|z_1-z_2|=\sqrt{|z_1|^2+|z_2|^2-2|z_1||z_1|\cos 60^\circ}$$
$$=\sqrt{4+9-6}=\sqrt{7}$$

$$\therefore \left|\frac{z_1-z_2}{z_1+z_2}\right|=\sqrt{\frac{7}{19}}$$

$$\therefore 19\left|\frac{z_1-z_2}{z_1+z_2}\right|^2=7$$

Matching Column Type

8. (c)

(p) Standard condition of equilateral triangle.

(q) $\text{Re}\left(\frac{z_3-z_1}{z_3-z_2}\right)=0\Rightarrow \arg\left(\frac{z_3-z_1}{z_3-z_2}\right)=\pm\frac{\pi}{2}$

$\Rightarrow$ Triangle is right angled.

(r) $\text{Re}\left(\frac{z_3-z_1}{z_3-z_2}\right)<0\Rightarrow \arg\left(\frac{z_3-z_1}{z_3-z_2}\right)>\frac{\pi}{2}$

(s) $\frac{z_3-z_1}{z_3-z_2}=i=e^{i\frac{\pi}{2}}$ and $|z_3-z_1|=|z_3-z_2|\Rightarrow$ triangle is isosceles and right angled.

Comprehension Type

9. (d) $|Z-2|\le|Z-4|\Rightarrow|Z-2|^2\le|Z-4|^2$

$\Rightarrow (x-2)^2+y^2\le(x-4)^2+y^2$

$\Rightarrow 8x-4x\le 16-4$

$\Rightarrow x\le 3$

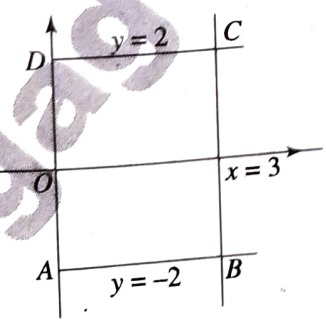
$|Z-3|\le|Z+3|\Rightarrow x\ge 0$

$|Z-i|\le|Z-3i|\Rightarrow y\le 2$

$|Z+i|\le|Z+3i|\Rightarrow y\ge -2$

So, z lies on or inside rectangle formed by lines $x=0$, $x=3$, $y=-2$ and $y=2$.

(1) Max of $|Z|=OC$ or $OB=\sqrt{3^2+2^2}=\sqrt{13}$



10. (a) $|Z_1-Z_2|_{max}=$ Length of diagonal

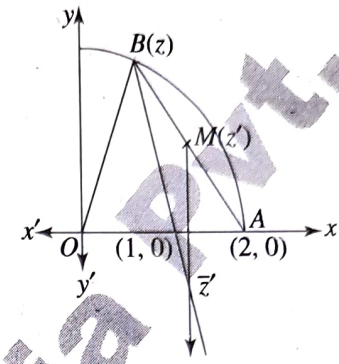
$=\sqrt{3^2+4^2}=5$

11. (a) $|Z_1-Z_2|_{min}=0$, occurs when z_1 and z_2 coincide.

DPP 3.7

Single Correct Answer Type

1. (a) $M(z')$ is mid-point of AB, so $z'=\frac{z+2}{2}$



$\Rightarrow \bar{z}'=\frac{\bar{z}+2}{2}$

Now z, 1, $\frac{\bar{z}}{2}+1$ are collinear

$$\Rightarrow \begin{vmatrix} z & \bar{z} & 1 \\ \frac{\bar{z}}{2}+1 & \frac{z}{2}+1 & 1 \\ 1 & 1 & 1 \end{vmatrix}=0$$

$$\Rightarrow z\left(\frac{z}{2}+1-1\right)-\bar{z}\left(\frac{\bar{z}}{2}+1-1\right)+1\left(\frac{\bar{z}}{2}-\frac{z}{2}\right)=0$$

$$\Rightarrow \frac{z^2}{2}-\frac{\bar{z}^2}{2}+\frac{(\bar{z}-z)}{2}=0$$

$$\Rightarrow (z-\bar{z})(z+\bar{z}-1)=0$$

$$\Rightarrow z-\bar{z}=0 \text{ or } (z+\bar{z}-1)=0$$

$$\Rightarrow z+\bar{z}=1 \text{ or } \text{Re}(z)=\frac{1}{2}$$

$$|z|=2\Rightarrow \frac{1}{4}+(\text{Im}(z))^2=4$$

$$\Rightarrow \text{Im}(z)=\frac{\sqrt{15}}{2}$$

$$\Rightarrow z=\frac{1}{2}+\frac{i\sqrt{15}}{2}$$

$$\Rightarrow \arg(z')=\tan^{-1}\left(\frac{\sqrt{15}}{5}\right)$$

2. (d) $|z_2+iz_1|=|z_1|+|z_2|\Rightarrow z_2$, iz_1, 0 are collinear

$\therefore \arg(iz_1)=\arg z_2$

$\Rightarrow \arg i+\arg z_1=\arg z_2$

$$\Rightarrow \arg z_2-\arg z_1=\frac{\pi}{2}$$

$$z_3=\frac{z_2-iz_1}{1-i}$$

$\Rightarrow (1-i)z_3=z_2-iz_1$

$\Rightarrow z_3-z_2=i(z_3-z_1)$

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$\therefore \quad \dfrac{z_3 - z_2}{z_3 - z_1} = i$

$\Rightarrow \quad \arg\left(\dfrac{z_3 - z_2}{z_3 - z_1}\right) = \dfrac{\pi}{2}$ and $|z_3 - z_2| = |z_3 - z_1|$

$\therefore \quad AC = BC$ and $AB^2 = AC^2 + BC^2$

$\therefore \quad 25 = 2AC^2$

$\Rightarrow \quad AC = \dfrac{5}{\sqrt{2}}$

Required area $= \dfrac{1}{2} \times \dfrac{5}{\sqrt{2}} \times \dfrac{5}{\sqrt{2}} = \dfrac{25}{4}$ sq. units

3. (a) Let A represents z_1.

Since $OA \cdot OB = 1 \quad \therefore |z_1 - 0| \cdot |z - 0| = 1$

$\Rightarrow \quad |z_1| = \dfrac{1}{|z|}$

Also, $\arg\left(\dfrac{z_1 - 0}{z - 0}\right) = 0 \Rightarrow \arg\left(\dfrac{z_1}{z}\right) = 0$

$\Rightarrow \quad \arg z_1 = \arg z$

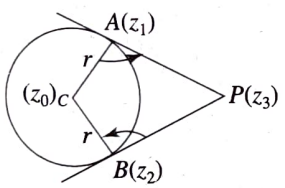
If θ is the argument of z, then

$z = |z| e^{i\theta}$

$\therefore \quad z_1 = \dfrac{1}{|z|} e^{i\theta} = \dfrac{1}{|z|^2} |z| e^{i\theta} = \dfrac{z}{z\bar{z}} = \dfrac{1}{\bar{z}}$

$\therefore \quad A$ is $\dfrac{1}{\bar{z}}$

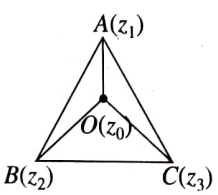
4. (b)



$\dfrac{z_3 - z_1}{z_0 - z_1} = \left(\dfrac{PA}{AC}\right) i$ and $\dfrac{z_0 - z_2}{z_3 - z_2} = \left(\dfrac{BC}{BP}\right) i$

$\therefore \quad \dfrac{(z_3 - z_1)(z_0 - z_2)}{(z_0 - z_1)(z_3 - z_2)} = \left(\dfrac{PA}{AC} \times \dfrac{BC}{PB}\right)(-1) = -1$

5. (c) Taking rotation at 'O'



$\dfrac{z_0 - z_1}{z_0 - z_2} = \cos 2C - i \sin 2C$

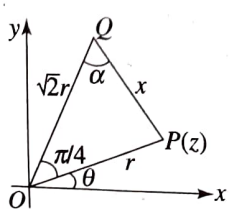
$\dfrac{z_0 - z_3}{z_0 - z_2} = \cos 2A + i \sin 2A$

Now $\left(\dfrac{z_0 - z_1}{z_0 - z_2}\right)\dfrac{\sin 2A}{\sin 2B} + \left(\dfrac{z_0 - z_3}{z_0 - z_2}\right)\dfrac{\sin 2C}{\sin 2B}$

$= \dfrac{\sin 2A \cos 2C - i \sin 2A \sin 2C + \cos 2A \sin 2C + i \sin 2A \sin 2C}{\sin 2B}$

$= \dfrac{\sin(2A + 2C)}{\sin 2B} = -1$

6. (c) $Z_P = r(\cos\theta + i \sin\theta)$



$Z_Q = \sqrt{2|z|^2}\left(\cos\left(\theta + \dfrac{\pi}{4}\right) + i \sin\left(\theta + \dfrac{\pi}{4}\right)\right)$

$= \sqrt{2}\, r\left[\cos\left(\theta + \dfrac{\pi}{4}\right) + i \sin\left(\theta + \dfrac{\pi}{4}\right)\right]$

From the figure,

$\cos\dfrac{\pi}{4} = \dfrac{2r^2 + r^2 - x^2}{2 \cdot \sqrt{2}r \cdot r} = \dfrac{3r^2 - x^2}{2\sqrt{2}r^2}$

$\therefore \quad 1 = \dfrac{3r^2 - x^2}{2r^2}$

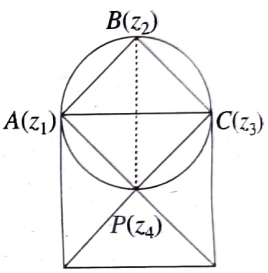
$\Rightarrow \quad r^2 = x^2 \Rightarrow x = r \Rightarrow$ Triangle is right isosceles.

Multiple Correct Answers Type

7. (c, d)

P is centre of square $\Rightarrow AP = PC$ and $\angle APC = \dfrac{\pi}{2}$

$\angle ABC$ is also $\dfrac{\pi}{2}$.



$\Rightarrow$ $ABCP$ is a cyclic quadrilateral.
$\Rightarrow$ A, B, C, P lie on a circle.
$\Rightarrow$ In circle chord, $AP = PC$
$\Rightarrow$ arc AP = arc PC
$\Rightarrow$ $\angle ABP = \angle PBC$
$\Rightarrow$ BP is angular bisector.

Let $\arg\left(\dfrac{z_1 - z_2}{z_4 - z_2}\right) = -\alpha$ (i)

$\Rightarrow \quad \arg\left(\dfrac{z_3 - z_2}{z_4 - z_2}\right) = \alpha$ (ii)

Add (i) and (ii), $\arg\left(\dfrac{z_1 - z_2}{z_4 - z_2}\right) + \arg\left(\dfrac{z_3 - z_2}{z_4 - z_2}\right) = 0$

Comprehension Type

8. (b), 9. (c), 10. (d)

We have $\alpha + \beta = -z_1$ and $\alpha\beta = z_2 + m$

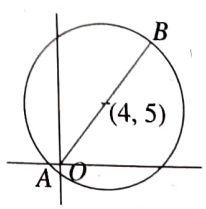
$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

$= z_1^2 - 4z_2 - 4m$

$= 16 + 20i - 4m$

Since $|\alpha - \beta| = 2\sqrt{7}$, we have $|4 + 5i - m| = 7$ (i)

Therefore, m lies on a circle having centre at (4, 5) and radius = 7.



$|m|_{max} = OB = 7 + \sqrt{41}$

Maximum argument of m is π which occurs at point A,

Let $m = x + iy$

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For point A, $m = x$

So from (i), $|4 + 5i - x| = 7$

or $(x-4)^2 = 24$

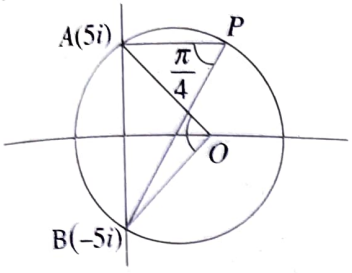
$\Rightarrow \quad x - 4 = \pm 2\sqrt{6}$

$\Rightarrow \quad x = 4 \pm 2\sqrt{6}$

For point A, $x = 2\sqrt{6} - 4$

11. (b), 12. (c), 13. (a)

11.



Given $\angle APB = \pi/4 \Rightarrow \angle AOB = \pi/2$

Let $OA = OB = R$ = radius of the circle

$\Rightarrow \quad 2R^2 = 10^2 \Rightarrow R = 5\sqrt{2}$

Length of the arc $= 5\sqrt{2} \times \dfrac{3\pi}{2}$

12. Centre (5, 0), radius $= 5\sqrt{2}$

$\therefore$ Equation of the circle $(x-5)^2 + y^2 = 50$

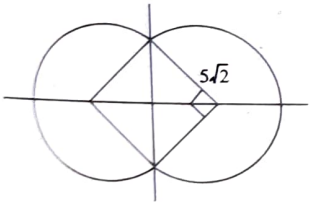
For $x = 1$, $y^2 = 34$.

Total number of integral points = (5 + 5 + 1).

Similarly for $x = 2, 3, \ldots 12$

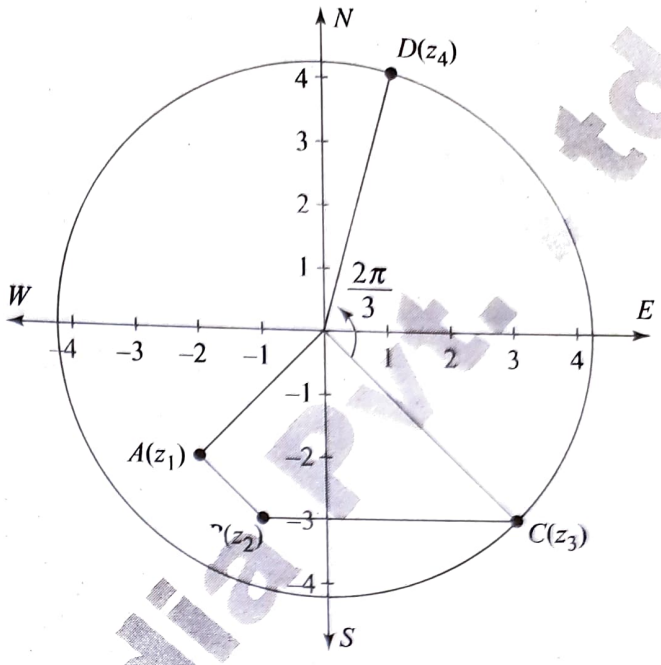
Finally number of integral points inside the region is 136.

13.



Required area $= 2\left[\dfrac{1}{2} \times (5\sqrt{2})^2 + \dfrac{1}{2}(5\sqrt{2})^2 \times \dfrac{3\pi}{2}\right]$

14. (c), 15.(c)



$Z_1 = 2\sqrt{2}\, e^{-i3\pi/4} = -2 - 2i$

$\dfrac{Z_2 - (-2-2i)}{O - (-2-2i)} = \dfrac{1}{2} \times e^{-i\pi/2}$ (rotation at A)

$\Rightarrow \quad Z_2 = -(1+i)(2-i) = -1 - 3i$

$\therefore \quad Z_3 = 3 - 3i$

$\dfrac{Z_4 - 0}{3 - 3i - 0} = e^{i2\pi/3} \Rightarrow Z_4 = 3(1-i)\omega$ (rotation at O)

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CHAPTER 4

DPP 4.1

Single Correct Answer Type

1. (a) The given result can be written as

$$\{x-(a-b)\}^2+\{x-(b-c)\}^2+(x-c)^2=0$$

$\Rightarrow \quad x=a-b \qquad x=b-c \qquad x=c$

$a-b=b-c=c$

From $a-b=b-c$, $2b=a+c$

$\therefore$ a, b, c can be in A.P.

2. (b) $y-x=3\left(\frac{1}{2^2}+\frac{1}{4^2}+\frac{1}{6^2}+\ldots\right)$

$x-z=\left(\frac{1}{2^2}+\frac{1}{4^2}+\frac{1}{6^2}+\ldots\right)$

$\therefore \quad (y-x)=3(x-z)$

$\Rightarrow \quad 4x=y+3z \Rightarrow 2\frac{x}{3}=\frac{y}{6}+\frac{z}{2}$

3. (b) Given $\frac{a+b}{1-ab}, b, \frac{b+c}{1-bc}$ are in A.P.

$$\Rightarrow \quad b-\frac{a+b}{1-ab}=\frac{b+c}{1-bc}-b$$

$$\Rightarrow \quad \frac{-a(b^2+1)}{1-ab}=\frac{c(b^2+1)}{1-bc}$$

$$\Rightarrow \quad a+c=2abc$$

Now, given quadratic equation is

$$2ac\,x^2+2abc\,x+2abc=0$$

(Substituting $a+c=2abc$ and then cancelling $2ac$)

$$\Rightarrow \quad x^2+bx+b=0$$

4. (a) As $1, a_1, a_2, a_3, \ldots, a_{87}, a_{88}, a_{89}, 89$ are in A.P.,

So, $a_1+a_{89}=a_2+a_{88}=\ldots=1+89=90$

$$\therefore \quad \sum_{r=1}^{89}\log(\tan a_r^\circ)$$

$$=\log(\tan a_1^\circ\cdot\tan a_2^\circ\ldots\tan a_{88}^\circ\cdot\tan a_{89}^\circ)$$

$$=\log 1=0$$

5. (c) $(a_{100}+b_{100})-(a_1+b_1)=99d \Rightarrow d=0$

$\therefore$ Sum of series $=(100/2)(a_{100}+b_{100}+a_1+b_1)$

$$=50(100+100)$$

$$=10000$$

6. (c) Let the first term be a and the common difference d.

Then it is given that $a+8d=55$ and (i)

$$1900<\frac{25}{2}(2a+24d)<2000$$

$\therefore \quad 1900<25(a+12d)<2000$

$\Rightarrow \quad 1900<25(a+8d)+25\times 4d<2000$

$\Rightarrow \quad 1900<25\times 55+100d<2000$

$$\Rightarrow \quad \frac{525}{100}<d<\frac{625}{100}$$

$\Rightarrow \quad d=6$ as d is an integer

$\therefore \quad a=55-8\times 6=7$

7. (a) Let common difference $= d$ and number of terms $= n$

$\therefore \quad T_5=m=l+4d \Rightarrow d=(m-l)/4$

$$\therefore \quad T_n=p=l+\frac{(n-1)(m-l)}{4}$$

$$\Rightarrow \quad n=\left(\frac{4p+m-5l}{m-l}\right)$$

$\therefore$ Sum of n terms of A.P. $=\frac{n}{2}$ [First term + Last term]

$$=\left[\frac{4p+m-5l}{m-l}\right]\cdot\frac{1}{2}[l+p] \qquad \text{(i)}$$

Comparing equation (i) with the given summation, we get $k=2$.

8. (a) Given A.P. is $a_1, a_2, a_3, \ldots, a_{15}$

$$a_1+a_{15}=a_2+a_{14}=\ldots=2a_8$$

$$a_1+a_{15}+a_8=\frac{3}{2}(a_1+a_{15})=15$$

$$\Rightarrow \quad a_1+a_{15}=10$$

$$a_2+a_3+a_8+a_{13}+a_{14}=2(a_1+a_{15})+a_8$$

$$=2(10)+5=25$$

9. (c) Let d be common difference of A.P. $\Rightarrow d=a_i-a_{i-1}$

Now $$\frac{1}{a_{i+1}^{2/3}+a_{i+1}^{1/3}\cdot a_i^{1/3}+a_i^{2/3}}=\frac{a_{i+1}^{1/3}-a_i^{1/3}}{a_{i+1}-a_i}$$

$$=\frac{1}{d}[a_{i+1}^{1/3}-a_i^{1/3}]$$

Thus $$\sum_{i=1}^{n}\frac{n}{a_{i+1}^{2/3}+a_{i+1}^{1/3}\cdot a_i^{1/3}+a_i^{2/3}}=\frac{1}{d}\sum_{i=1}^{n-1}(a_{i+1}^{1/3}-a_i^{1/3})$$

$$=\frac{1}{d}(a_n^{1/3}-a_1^{1/3})$$

$$=\frac{1}{d}\frac{(a_n-a_1)}{a_n^{2/3}+a_n^{1/3}\cdot a_1^{1/3}+a_1^{2/3}}$$

$$=\frac{(n-1)}{a_n^{2/3}+a_n^{1/3}\cdot a_1^{1/3}+a_1^{2/3}}$$

10. (a) $a, A_1, A_2, \ldots A_n, b$ are in A.P.

where $a=2, b=20, n=8$

$\therefore$ Sum of the means $=\frac{n}{2}(a+b)=\frac{8}{2}(2+20)=88$

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11. (b) $\frac{a_2-a_1}{a_1a_2}+\frac{a_3-a_2}{a_2a_3}+\ldots+\frac{a_{4001}-a_{4000}}{a_{4001}a_{4000}}=10d$

$\therefore \quad \frac{1}{a_1}-\frac{1}{a_{4001}}=10d$

$\therefore \quad \frac{4000d}{a_1a_{4001}}=10d$

$\therefore \quad a_1a_{4001}=400$

also $a_1+a_{4001}=50$

$\Rightarrow \quad |a_1-a_{4001}|^2=2500-1600$

$\Rightarrow \quad |a_1-a_{4001}|=30$

12. (d) Given mid terms $t_n=1$ and $t_{n+1}=7$

$\therefore \quad t_n+t_{n+1}=8=t_1+t_{2n}$

and $t_{n+1}-t_n=6=d$ (common difference of A.P.)

$t_n+t_{n+1}=8$

$\therefore \quad a+(n-1)d+a+nd=8$

$\therefore \quad a+6n=7$

Now $4t_1t_{2n}=[(t_1+t_{2n})^2-(t_{2n}-t_1)^2]$

$=64-36(2n-1)^2 \qquad$ [as $t_{2n}-t_1=(2n-1)\times 6$]

$\therefore \quad t_1t_{2n}=16-9(2n-1)^2$

$\therefore \quad 16-9(2n-1)^2+713\geq 0$

$\therefore \quad -4\leq n\leq 5$

$\therefore \quad n=5$

Hence, from $a+6n=7$, $a=-23$

13. (d)

$x_1+x_2+x_3+\ldots+x_{100}=\frac{100}{2}(x_1+x_{100})=-1$

$\Rightarrow \quad x_1+x_{100}=-\frac{1}{50}$

$x_2+x_4+\ldots+x_{100}=\frac{50}{2}(x_1+d+x_{100})=1$

$\Rightarrow \quad x_1+x_{100}+d=\frac{1}{25}$

$\Rightarrow \quad d=\frac{3}{50}$

$x_1+x_1+99d=\frac{-1}{50}$

$\Rightarrow \quad x_1=\frac{-149}{50}$

$x_{100}=x_1+99d$

$=\frac{-149}{50}+99\times\frac{3}{50}$

$=\frac{74}{25}$

14. (a) Given $\frac{x_1}{x_1+1}=\frac{x_2}{x_2+3}=\frac{x_3}{x_3+5}=\ldots=\frac{x_{2005}}{x_{2005}+4009}=\lambda$ (say)

$\Rightarrow \quad x_1=\frac{\lambda}{1-\lambda}, x_2=\frac{3\lambda}{1-\lambda}, x_3=\frac{5\lambda}{1-\lambda}, \ldots$

Hence, $x_1, x_2, x_3, \ldots, x_{2005}$ are in arithmetic progression.

DPP 4.2

Single Correct Answer Type

1. (a) $(bx-cy)^2=(b-c)(bx^2-cy^2)$

$b^2x^2+c^2y^2-2bxy=b^2x^2-bcy^2-bcx^2+c^2y^2$

$bcx^2+bcy^2-2bcxy=0$

$bc(x-y)^2=0$

$\Rightarrow \quad x=y \qquad (\because b, c\neq 0)$

$\Rightarrow \quad \left(\frac{bx+cy}{b+c}\right)\left(\frac{bx-cy}{b-c}\right)=x^2$

2. (c) We have $a_1+a_3+a_5+\ldots+a_{2n-1}$

$=a_5+a_7+a_9+\ldots+a_{2n+3}$

$=r^4(a_1+a_3+\ldots+a_{2n-1})$

$\Rightarrow \quad r^4=1$

$\Rightarrow \quad r=1, -1, i$ and $-i$.

3. (b) $\therefore \quad (2+b)^2=(3+a)(3+c)$

$\therefore \quad 4+b^2+4b=9+3(a+c)+ac$

$\therefore \quad 4+b^2+4b=9+6b+ac$

$\therefore \quad ac=b^2-2b-5=(b-1)^2-6$

$\therefore \quad ac\geq -6$

4. (d) Let r is the common ratio.

$\Rightarrow \quad \frac{a+ar}{2}=6$ and $\frac{ar^2+ar^3}{2}=54$

$\Rightarrow \quad r^2=9\Rightarrow r=\pm 3\Rightarrow r=3(r\neq -3)$

When $r=3$, $a=3$, AM of a and $d=\frac{a+ar^3}{2}=42$

5. (b) $a+b+c=60$ and $2b=a+c$

$\therefore \quad b=20$

$\Rightarrow \quad c=40-a$

$\therefore \quad a-2, 20, 43-a$ are in G.P.

$\Rightarrow \quad (a-2)(43-a)=400$

$\Rightarrow \quad a=27$ or 18

If $a=27$, then $b=20$, $c=13$

If $a=18$, then $b=20$, $c=22$

6. (d) $\log_6(abc)=6$

$\Rightarrow \quad (abc)=6^6$

Let $a=\frac{b}{r}$ and $c=br$

$\Rightarrow \quad b=36$ and $a=\frac{36}{r}\Rightarrow r=2, 3, 4, 6, 9, 12, 18$

Also $36\left(1-\frac{1}{r}\right)$ is a perfect cube.

$\Rightarrow \quad r=4$

$\Rightarrow \quad a+b+c=9+36+144=189.$

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7. (c) The three terms of the geometric sequence with the common ratio r are x, xr, xr^2.

$\therefore \quad x + xr + xr^2 = 42$

After multiplying the middle term by 5/4, we will get an arithmetic sequence.

$\therefore \quad \frac{5}{4}xr - x = xr^2 - \frac{5}{4}xr$

$\therefore \quad 2r^2 - 5r + 2 = 0, \qquad \therefore \quad r = \frac{1}{2}$ and $r = 2$.

On substituting these in $x + xr + xr^2 = 42$, we get

$x = 6$ or 24

8. (c) Sum $= \frac{\frac{x}{r}}{1-r} = 4$ (where r is common ratio)

$x = 4r(1 - r)$

$= 4(r - r^2)$

For $r \in (-1, 1)$

$r - r^2 \in \left(-2, \frac{1}{4}\right]$

$\Rightarrow \quad x \in (-8, 1]$

9. (b) Let G.P. be $a, ar, ar^3, \ldots, ar^{n-1}$

$$S_{11} = \frac{a(1-r^{11})}{1-r}$$

$$S'_{11}(\text{from last}) = ar^{n-11}\frac{(1-r^{11})}{1-r}$$

$$\Rightarrow \quad \frac{S_{11}}{S'_{11}} = \frac{1}{r^{n-11}}$$

Given $\frac{1}{r^{n-11}} = \frac{1}{8}$

$\Rightarrow \quad r^{n-11} = 8$

Also for $ar^9 \ldots ar^{n-1}$, $S = ar^9 \cdot \frac{(1-r^{n-9})}{1-r}$,

and S'(sum of all the terms without the last nine) $= \frac{a \cdot (1-r^{n-9})}{1-r}$

Now $\frac{S}{S'} = r^9 = 2$ (given)

$\Rightarrow \quad r^{n-11} = 2^3 = (r^9)^3$

$\Rightarrow \quad n - 11 = 27$

$\Rightarrow \quad n = 38$

Comprehension Type

10. (c), 11. (a)

$a, b, a^2, \ldots$ is an A.P.

$\therefore \quad 2b = a^2 + a$ (i)

$a, a^2, b \ldots$ is a G.P.

$\therefore \quad a^3 = b$ (ii)

$\therefore \quad a^2 + a = 2a^3$

$\therefore \quad a = 0, a = 1$ or $a = -1/2$

$\therefore \quad a = -1/2$ (as $a < 0$)

$\therefore \quad b = -\frac{1}{8}$

In G.P. putting $a = -\frac{1}{2}$ and $b = -\frac{1}{8}$

$\therefore$ G.P. is $-\frac{1}{2}, \frac{1}{4}, -\frac{1}{8} \ldots$

Sum of infinite G.P. $= \frac{a}{1-r} = \frac{\left(-\frac{1}{2}\right)}{1-\left(\frac{\frac{1}{4}}{\frac{-1}{2}}\right)} = \frac{-1}{3}$

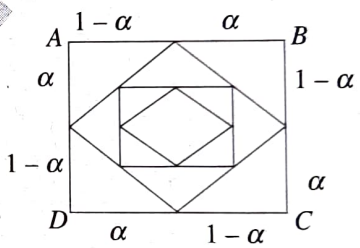
Sum of 40 terms of A.P. $= \frac{n}{2}[2a + (n-1)d]$

$$= \frac{40}{2}\left[2\left(\frac{-1}{2}\right) + (40-1)\times\left(-\frac{1}{8} + \frac{1}{2}\right)\right]$$

$$= 20\left[-1 + 39 \times \frac{3}{8}\right]$$

$$= \frac{545}{2}$$

12. (b), 13. (b), 14. (c)



Clearly from the diagram,

$a_1 = 1$

$(a_{n+1})^2 = [(1-\alpha)^2 + \alpha^2](a_n)^2 \Rightarrow A_{n+1} = [2\alpha^2 - 2\alpha + 1]A_n$

For $\alpha = \frac{1}{3}$, $(a_{n+1})^2 = \frac{5}{9}(a_n)^2$

$\Rightarrow \quad \frac{A_{n+1}}{A_n} = \frac{5}{9}$

$\Rightarrow \quad A_2 = \frac{5}{9}, A_3 = \left(\frac{5}{9}\right)^2, \ldots$

$\Rightarrow \quad A_n = \left(\frac{5}{9}\right)^{n-1}$

$A_n < \frac{1}{10} \Rightarrow \left(\frac{5}{9}\right)^{n-1} < \frac{1}{10}$

$\Rightarrow$ Least value of n is 5.

$\sum_{n=1}^{\infty} A_n = \frac{8}{3}$

$\Rightarrow \quad \frac{1}{1-(2\alpha^2 - 2\alpha + 1)} = \frac{8}{3}$

$\Rightarrow \quad 16\alpha^2 - 16\alpha + 3 = 0$

$\Rightarrow \quad 16\alpha^2 - 12\alpha - 4\alpha + 3 = 0$

$\Rightarrow \quad (4\alpha - 1)(4\alpha - 3) = 0$

$\Rightarrow \quad \alpha = \frac{1}{4}, \frac{3}{4}$

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The side of n^{th} square equal to the diagonal of $(n+1)^{th}$ square.

$\Rightarrow \sqrt{2}a_n = a_{n+1}$

$\Rightarrow \frac{(a_{n+1})^2}{(a_n)^2} = \frac{1}{2}$

$\Rightarrow 4\alpha^2 - 4\alpha + 1 = 0$

$\Rightarrow \alpha = 1/2$

DPP 4.3

Single Correct Answer Type

1. (c) 4, x, y are in H.P.

$$\frac{2}{x} = \frac{1}{4} + \frac{1}{y}$$

$$\Rightarrow \frac{2}{x} - \frac{1}{4} = \frac{1}{y}$$

$$\Rightarrow \frac{8-x}{4x} = \frac{1}{y}$$

$$\Rightarrow y = \frac{4x}{8-x} = \frac{4(8-(8-x))}{8-x} = \frac{32}{8-x} - 4$$

$8 - x$ must be a factor of 32

$8 - x = 1 \Rightarrow x = 7, y = 28$

$8 - x = 2 \Rightarrow x = 6, y = 12$

$8 - x = 4 \Rightarrow x = 4, y = 4$

$8 - x = 8 \Rightarrow x = 0, y = 0$ (Not possible)

$\therefore$ Number of ordered pairs of (x, y) is 3.

2. (b) $a + c, a + b, a + c$ are in G.P.

$\therefore (a+b)^2 = (a+c)(b+c)$

$(a+b)^2 = ab + c(a+b) + c^2$ (1)

a, c, b are in H.P.

$$\therefore c = \frac{2ab}{a+b} \quad (2)$$

From (1) and (2),

$$(a+b)^2 = \frac{3}{2}c(a+b) + c^2$$

$\therefore 2(a+b)^2 - 3c(a+b) - 2c^2 = 0$

$\therefore a + b = \frac{-c}{2}$ (rejected)

or $a + b = 2c$

$\therefore \frac{a+b}{c} = 2$

3. (a) a, b, c are in H.P. $\Rightarrow b = \frac{2ac}{a+c}$ (i)

b, c, d are in G.P. $\Rightarrow c^2 = bd$ (ii)

c, d, e are in A.P. $\Rightarrow d = \frac{c+e}{2}$ (iii)

From (i), $ab + bc = 2ac$ (iv)

$$\Rightarrow c = \frac{ab}{2a-b}$$

From (iii) and (iv),

$$d = \frac{1}{2}\left[\frac{ab}{2a-b} + e\right] \quad (v)$$

From (ii), (iv) and (v),

$$\frac{a^2b^2}{(2a-b)^2} = \frac{b}{2}\left[\frac{ab}{2a-b} + e\right]$$

$$\Rightarrow e = \frac{ab^2}{(2a-b)^2}$$

4. (b) Let $x = e, y = e^2, z = e^3$

$\therefore$ Given terms are $\log_{e^2} e, \log_{e^3} e, \log_{e^4} e$

or $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ which are in H.P.

5. (c) As x, y, z are in G.P.

$\Rightarrow e^{2x}, e^{4x}, e^{6x}$ are in G.P.

$\Rightarrow \log_e(e^{2x}), \log_e(e^{4x}), \log_e(e^{6x})$ are in A.P.

$$\Rightarrow \frac{1}{2x + \log_e x}, \frac{1}{4x + \log_e y}, \frac{1}{6x + \log_e z} \text{ are in H.P.}$$

6. (b) $a + b = 12$

$$ab + \frac{6ab}{a+b} = 48$$

$$\therefore ab + \frac{ab}{2} = 48$$

$$\therefore ab = 32$$

7. (b) Let $x = \frac{k}{r}, y = k, z = kr$

$$\Rightarrow \frac{\frac{k}{r} + k}{2} = 4 \quad (i)$$

and $\frac{2k \cdot kr}{k + kr} = 9$ (ii)

Solving we get $k = 6$

8. (b) Given quantities are $\frac{1}{2}, \frac{1}{2^2}, \frac{1}{2^3}, \ldots, \frac{1}{2^{10}}$

$$\text{H.M.} = \frac{10}{2 + 2^2 + 2^3 + \ldots + 2^{10}}$$

$$= \frac{10}{\frac{2(2^{10}-1)}{2-1}} = \frac{5}{2^{10}-1}$$

$\therefore \lambda = 5$

9. (c) Average speed = H.M

$$\frac{4}{\frac{1}{100} + \frac{1}{200} + \frac{1}{300} + \frac{1}{400}} = 192$$

Multiple Correct Answers Type

10. (a, b, c)

$2x = a + b$

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$y^2 = ab$

$z = \frac{2ab}{a+b}$

$x = y + 2$

and $a = 5z$

$z = \frac{2y^2}{2x}$

$\Rightarrow \quad y^2 = xz$

$x = y + 2$

$\therefore \quad \frac{a+b}{2} = \sqrt{ab} + 2$ (i)

and $a = 5\frac{(2ab)}{a+b}$

$\Rightarrow \quad (a+b) = 10b$

$\Rightarrow \quad a = 9b$

$\therefore$ from (i) $\frac{9b+b}{2} = \sqrt{9b^2} + 2$

$\Rightarrow \quad 5b = 3b + 2$

$\Rightarrow \quad b = 1$

So $a = 9$

$\Rightarrow \quad x > y > z$

11. (a, b)

We have $A_1 = \frac{3a+b}{4}, A_2 = \frac{a+b}{2}, A_3 = \frac{a+3b}{4}$

$G_1 = (a^3b)^{1/4}, G_2 = (ab)^{1/2}, G_3 = (ab^3)^{1/4},$

$H_1 = \frac{4ab}{(a+3b)}, H_2 = \frac{2ab}{(a+b)}, H_3 = \frac{4ab}{(3a+b)}$

$\Rightarrow \quad A_2H_2 = ab = G_2^2$

$G_2^2 = A_1H_3 = A_2H_2 = A_3H_1 = ab$

12. (b, c)

$\alpha, \beta, \gamma, \delta$ are in H.P.

$\Rightarrow \quad \frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}, \frac{1}{\delta}$ are in A.P.

Let d be the common difference of the A.P.

Since α, γ are roots of $Ax^2 - 4x + 1 = 0$

$\therefore \quad \frac{\alpha+\gamma}{\alpha\gamma} = \frac{4/A}{1/A} = 4$

or $\frac{1}{\alpha} + \frac{1}{\gamma} = 4$

$\Rightarrow \quad \frac{1}{\alpha} + \frac{1}{\alpha} + 2d = 4$ or $\frac{1}{\alpha} + d = 2$ (i)

Also β, δ are roots of $Bx^2 - 6x + 1 = 0$

$\therefore \quad \frac{\beta+\delta}{\beta\delta} = \frac{1}{\beta} + \frac{1}{\delta} = \frac{6/B}{1/B} = 6$ or $\frac{1}{\alpha} + d + \frac{1}{\alpha} + 3d = 6$

$\Rightarrow \quad \frac{1}{\alpha} + 2d = 3$ (ii)

Solving (i) and (ii), we get $\frac{1}{\alpha} = 1$ and $d = 1$.

$\therefore \quad \frac{1}{\alpha} = 1, \frac{1}{\beta} = 2, \frac{1}{\gamma} = 3$ and $\frac{1}{\delta} = 4.$

Since, $\frac{1}{\alpha\gamma} = A \Rightarrow A = 3.$

Also, $\frac{1}{\beta\gamma} = B \Rightarrow B = 8.$

13. (a, d) $\frac{1}{a} + \frac{1}{c} = \frac{1}{2b-a} + \frac{1}{2b-c}$

$\Rightarrow \quad \frac{1}{a} + \frac{1}{a-2b} + \frac{1}{c} + \frac{1}{c-2b} = 0$

$\Rightarrow \quad \left(\frac{1}{a} + \frac{1}{c-2b}\right) + \left(\frac{1}{a-2b} + \frac{1}{c}\right) = 0$

$\Rightarrow \quad (a+c-2b)\left(\frac{1}{a(c-2b)} + \frac{1}{c(a-2b)}\right) = 0$

Either, $a + c - 2b = 0$

$\Rightarrow \quad a, b, c$ are in A.P. or $\frac{1}{a(c-2b)} + \frac{1}{c(a-2b)} = 0$

$\Rightarrow \quad ac - 2bc + ac - 2ab = 0$

$\Rightarrow \quad 2ac = 2b(a+c)$

$\Rightarrow \quad b = \frac{ac}{a+c} \Rightarrow a, 2b, c$ are in H.P.

DPP 4.4

Single Correct Answer Type

1. (a) The given series can be written as $1^3 + \frac{1^3+2^3}{1+3} + \frac{1^3+2^3+3^3}{1+3+5} + \ldots$

$t_n = \frac{1^3+2^3+3^3+\ldots..n^3}{1+3+5+\ldots.+(2n-1)}$

$= \frac{n^2(n+1)^2}{4n^2} = \frac{(n+1)^2}{4}$

$= \frac{1}{4}(n^2+2n+1)$

$\therefore \quad S_n = \frac{1}{4}\left[\sum_{k=1}^{n} k^2 + 2\sum_{k=1}^{n} k + n\right]$

$\therefore \quad S_n = \frac{1}{4}\left[\frac{n(n+1)(2n+1)}{6} + n(n+1) + n\right]$

$\therefore \quad S_{16} = \frac{1}{4}\left[\frac{16.17.33}{6} + 16.17 + 16\right]$

$= \frac{1}{4}[88 \times 17 + 16 \times 17 + 16] = 446$

2. (b) $S = 2 \times 5 + 5 \times 9 + 8 \times 13 + \ldots$ 10 terms

$t_r = (3r-1)(4r+1)$

$= 12r^2 - r - 1$

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$$\therefore S=\sum_{r=1}^{10}(12r^2-r-1)$$

$$=12\frac{10\cdot11\cdot21}{6}-\frac{10\cdot11}{2}-10$$

$$=10\left(462-\frac{11}{2}-1\right)$$

$$=4555$$

3.(c) $S=ab+[ab+(a+b)+1]+[ab+2(a+b)+2^2]$
$+\ldots+[ab+(n-1)(a+b)+(n-1)^2]$

$$=nab+(a+b)\sum_{r=1}^{n-1}r+\sum_{r=1}^{n-1}r^2$$

$$=nab+(a+b)\frac{n(n-1)}{2}+\frac{(n-1))(n)(2n-1)}{6}$$

$$=\frac{n}{6}[1+(n-1)\{1+2n-1\}]$$

$$=\frac{n}{6}[1+2n(n-1)]=\frac{n}{6}(1-2n+2n^2)$$

4.(a) $S=\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\frac{1}{a^{i+j+k}}, |a|>1$ or $0<\frac{1}{|a|}<1$

$$=\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\frac{1}{a^ia^ja^k}$$

$$=\left(\sum_{i=1}^{\infty}\frac{1}{a^i}\right)\left(\sum_{j=1}^{\infty}\frac{1}{a^j}\right)\left(\sum_{k=1}^{\infty}\frac{1}{a^k}\right)$$

$$=\frac{\frac{1}{a}}{1-\frac{1}{a}}\cdot\frac{\frac{1}{a}}{1-\frac{1}{a}}\cdot\frac{\frac{1}{a}}{1-\frac{1}{a}}=\frac{1}{(a-1)^3}$$

5.(b) $(x+1)(x-2)^2(x+3)^3(x-4)^4\ldots(x+49)^{49}(x-50)^{50}$

Degree of the expression is $1+2+3+\ldots+50=1275$

Coefficient of x^{1274} is $(1-2-2+3+3+3-4-4-4-4\ldots)$

$$=1^2-2^2+3^2-4^2+\ldots+49^2-50^2$$

$$=-(1+2+3+\ldots+50)$$

$$=-1275$$

6.(c) Let 2010 be in k^{th} row

$\Rightarrow$ k^{th} term of series $1, 2, 4, 7, \ldots \le 2010$ (series formed by 1st term of each group)

and $(k+1)^{th}$ term of series $1, 2, 4, 7, \ldots > 2010$

$$\begin{array}{ll} S_n= & 1+2+4+7+\ldots+T_n \\ S_n= & 1+2+4+\ldots+T_{n-1}+T_n \\ \hline 0= & 1+(1+2+3+\ldots(n-1)\text{ terms})-T_n \end{array}$$

$$\Rightarrow T_n=\frac{n^2-n+2}{2}$$

$$\Rightarrow \frac{k^2-k+2}{2}\le 2010 \text{ and } \frac{k^2+k+2}{2}>2010$$

$$\Rightarrow k^2-k-4018\le 0 \text{ and } k^2+k-4018>0$$

$$\Rightarrow \left(k-\frac{1}{2}\right)^2\le\frac{16073}{4} \text{ and } \left(k+\frac{1}{2}\right)^2>\frac{16073}{4}$$

$$\Rightarrow k-\frac{1}{2}\le 63.3 \text{ and } k+\frac{1}{2}>63.3$$

$$\Rightarrow k=63$$

7. (c) $\sum_{i=1}^{n}\sum_{j=1}^{i}\sum_{k=1}^{j}1$

$$=\sum_{i=1}^{n}\sum_{j=1}^{i}j$$

$$=\sum_{i=1}^{n}\frac{i(i+1)}{2}$$

$$=\frac{1}{2}\sum_{i=1}^{n}(i^2+i)$$

$$=\frac{n(n+1)(n+2)}{6}=220$$

$$\therefore n=10$$

8. (b) $\sum_{\substack{k=1\\ i\ne j\ne k}}^{10}\sum_{j=1}^{10}\sum_{i=1}^{10}1$

= (sum when i, j, k are independent)
– (sum when any two of i, j, k are equal)
– (sum when all i, j, k are equal)

$$=\sum_{\substack{k=1\\ i\ne j\ne k}}^{10}\sum_{j=1}^{10}\sum_{i=1}^{10}1-3\sum_{\substack{k=1\\ i=j}}^{10}\sum_{i=1}^{10}1+2\sum_{\substack{k=1\\ i=j=k}}^{10}1$$

$$=10^3-3\times10^2+2\times10$$

$$=720$$

9. (a) $\sum_{\substack{k=1\\ i<j<k}}^{10}\sum_{j=1}^{10}\sum_{i=1}^{10}1$

$$=\frac{1}{6}\sum_{\substack{k=1\\ i\ne j\ne k}}^{10}\sum_{j=1}^{10}\sum_{i=1}^{10}1$$

As in $\sum_{\substack{k=1\\ i<j<k}}^{10}\sum_{j=1}^{10}\sum_{i=1}^{10}1$, we have sum of terms for $i<j<k$, $i<k<j$, $j<i<k, j<i<k, k<i<j, k<j<i$ and sum for each inequality is same

$$\therefore \sum_{\substack{k=1\\ i<j<k}}^{10}\sum_{j=1}^{10}\sum_{i=1}^{10}1$$

$$=\frac{1}{6}\sum_{\substack{k=1\\ i\ne j\ne k}}^{10}\sum_{j=1}^{10}\sum_{i=1}^{10}1$$

$$=\frac{720}{6}$$

$$=120$$

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Comprehension Type

10. (b), 11. (b)

$$\begin{aligned} S &= 3 + 6 + 11 + 18 + \ldots + t_n \\ S &= \quad 3 + 6 + 11 + \ldots + t_{n-1} + t_n \end{aligned}$$

$$\therefore \quad 0 = 3 + (3 + 5 + 7 + 9 + \ldots (n-1) \text{ terms}) - t_n$$

$$\therefore \quad t_n = n^2 + 2$$

Similarly nth terms of $g(n) = n^2 + n + 1$

$$\therefore \quad \lim_{n\to\infty} \frac{n^2+2}{n^2+n+1} = 1$$

$$F(n) = \sum (n^2 + 2)$$

$$= \frac{n(n+1)(2n+1)}{6} + 2n$$

$$= \frac{n(2n^2+3n+13)}{6}$$

$$G(n) = \sum (n^2 + n + 1) = \frac{n(n^2+3n+5)}{3}$$

$$\therefore \quad \lim_{n\to\infty} \frac{\frac{n(2n^2+3n+13)}{6}}{\frac{n(n^2+3n+5)}{3}} = 1$$

DPP 4.5

Single Correct Answer Type

1. (b) $S = 1 + 4x + 7x^2 + 10x^3 + \ldots$

$x.S = x + 4x^2 + 7x^3 + \ldots$

Subtracting

$$S(1-x) = 1 + 3x + 3x^2 + 3x^3 + \ldots$$

$$S(1-x) = 1 + 3x\left(\frac{1}{1-x}\right) \qquad |x| < 1$$

$$\therefore \quad S = \frac{1+2x}{(1-x)^2}$$

Given $\qquad \frac{1+2x}{(1-x)^2} = \frac{35}{16}$

$$\Rightarrow \quad 16 + 32x = 35 + 35x^2 - 70x$$

$$\Rightarrow \quad (5x-1)(7x-19) = 0$$

But $|x| < 1 \therefore x = \frac{1}{5}$

2. (c) We have $S = \frac{1}{5} - \frac{2}{5^2} + \frac{3}{5^3} - \frac{4}{5^4} + \frac{5}{5^5} + \ldots \infty$

$$\frac{S}{5} = \frac{1}{5^2} - \frac{2}{5^3} + \frac{3}{5^4} - \frac{4}{5^5} + \ldots \infty$$

Adding

$$\frac{6S}{5} = \frac{1}{5} - \frac{1}{5^2} + \frac{1}{5^3} - \frac{1}{5^4} - \frac{1}{5^5} - \ldots \infty$$

$$\frac{6S}{5} = \frac{\frac{1}{5}}{1+\frac{1}{5}} = \left(\frac{1}{5}\right)\left(\frac{5}{6}\right) \Rightarrow S = \frac{5}{36}$$

3. (a) $T_n = \frac{1}{3}\left[\frac{1}{3} + \frac{1}{6} + \frac{1}{10} + \frac{1}{15} + \frac{1}{21} + \ldots\right]$

$$= \frac{2}{3}\left[\frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \frac{1}{4\cdot 5} + \frac{1}{5\cdot 6} + \ldots\right]$$

Hence $T_n = \frac{2}{3}\frac{1}{(n+1)(n+2)}$

$$= \frac{2}{3}\left[\frac{1}{n+1} - \frac{1}{n+2}\right]$$

$$\therefore \quad S_\infty = \frac{2}{3}\cdot\frac{1}{2} = \frac{1}{3}$$

4. (c) $T_e = \frac{r^4+r^2+1}{r^4+r} = \frac{(r^2+r+1)(r^2-r+1)}{r(r+1)(r^2-r+1)} = \frac{r^2+r+1}{r(r+1)}$

$$T_r = 1 + \frac{1}{r} - \frac{1}{r+1}$$

$$T_1 = 1 + \frac{1}{1} - \frac{1}{2}$$

$$T_2 = 1 + \frac{1}{2} - \frac{1}{3}$$

$$T_3 = 1 + \frac{1}{3} - \frac{1}{4}$$

.....

$$T_n = 1 + \frac{1}{n} - \frac{1}{n+1}$$

$$\therefore \quad S_n = n + 1 - \frac{1}{n+1} = \frac{675}{26}$$

$$\therefore \quad 26(n+1)^2 - 26 = 675(n+1)$$

$$\Rightarrow \quad 26(n+1)^2 - 675(n+1) - 26 = 0$$

$$\Rightarrow \quad 26(n+1)[n+1-26] + [(n+1)-26] = 0$$

$$\Rightarrow \quad (n-25)(26n+27) = 0$$

$$\therefore \quad n = 25$$

5. (d) $\frac{1}{x_{k+1}} = \frac{1}{x_k(x_k+1)} = \frac{1}{x_k} - \frac{1}{x_k+1}$

$$\Rightarrow \quad \frac{1}{x_k+1} = \frac{1}{x_k} - \frac{1}{x_{k-1}}$$

$$\therefore \quad \frac{1}{x_1+1} + \frac{1}{x_2+1} + \ldots + \frac{1}{x_{100}+1} = \frac{1}{x_1} - \frac{1}{x_{101}}$$

As $0 < \frac{1}{x_{101}} < 1$

$$\therefore \quad \left[\frac{1}{x_1+1} + \frac{1}{x_2+1} + \ldots + \frac{1}{x_{100}+1}\right] = 1$$

6. (b) We have $S_n - S_{n-2} = T_n + T_{n-1}$ (Taking n to be odd)

$$\therefore \quad T_{n-1}\left(1 + \frac{T_n}{T_{n-1}}\right) = \left(\frac{n+1}{2}\right) - \frac{(n-1)}{2} = 1$$

$$\left(\text{As } S_n = \frac{n+1}{2}, \text{ if } n \text{ is odd}\right)$$

$$\therefore \quad T_{n-1}\left(1 + \frac{n^2}{1-n^2}\right) = 1$$

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$\therefore\ T_{n-1} = -(n^2-1)$ when n is odd

Also, $S_m = S_{m-1} + T_m$

If m is even then $(m-1)$ is odd.

$\therefore\ S_m = \frac{m}{2} - ((m+1)^2 - 1) = \frac{-m(2m+3)}{2}$

$\therefore\ |S_{20}| = \left|\frac{-20}{2}(43)\right| = 430$

7. (b) $T_n = (n^2 - n + 1)n!$

$= (n^2-1)n! - (n-2)n!$

$T_n = (n-1)(n+1)! - (n-2)n!$

$\therefore\ S_n = 1 + (n-1)(n+1)!$

$\therefore\ S_{50} = 1 + 49 \times 51!$

8. (d) Here $t_r = \frac{r \cdot 2^r}{(r+2)!} = \frac{(r+2-2)2^r}{(r+2)!} = \frac{2^r}{(r+1)!} - \frac{2^{r+1}}{(r+2)!}$

$\therefore\ S_n = \sum_{r=1}^{n} t_r = \sum_{r=1}^{n} \frac{2^r}{(r+1)!} - \frac{2^{r+1}}{(r+2)!} = 1 - \frac{2^{n+1}}{(n+2)!}$

$\therefore\ S_\infty = \lim_{n\to\infty}\left(1 - \frac{2^{n+1}}{(n+2)!}\right) = 1$

9. (b) Sequence is $t_1 + t_2 + t_3 + t_4 + \ldots$

$t_3 = t_1 + t_2,\ t_7 = 1000$

$t_1 = 1$

but $t_7 = t_1 + t_2 + t_3 + t_4 + t_5 + t_6$

$1000 = 2(t_1 + t_2 + t_3 + t_4 + t_5)$

$= 4(t_1 + t_2 + t_3 + t_4)$

$= 8(t_1 + t_2 + t_3)$

$= 16(t_1 + t_2)$

$\therefore\ t_1 + t_2 = 125/2$

$\therefore\ t_2 = 125/2 - 1 = 123/2$

10. (b) We have $x_k + k = S - x_k$ where $x_1 + x_2 + \ldots + x_k = S$

$\Rightarrow\ 2x_k + k = S$

$\therefore\ 2\sum_{k=1}^{50} x_k + \sum_{k=1}^{50} k = \sum_{k=1}^{50} S$

$\Rightarrow\ 2(S) + \frac{50.51}{2} = 50S$

$\Rightarrow\ 48(S) = 25.51$

$\Rightarrow\ x_{20} = \left(\frac{25.51}{48} - 20\right)\frac{1}{2} = \frac{315}{96}.$

11. (a) $a_n = 3a_{n-1} + 1$

$a_{2010} = 3a_{2009} + 1$

$= 3(3a_{2008} + 1) + 1 = 3^2 a_{2008} + 3 + 1$

$= 3^3 a_{2007} + 3 + 3 + 1$

$= 3^4 a_{2006} + 3 + 3 + 3 + 1$

$= 3^{2010} a_0 + (3 + 3 + 3 + \ldots \text{ 2009 times}) + 1$

$= 3 \times 2009 + 1$

$= 6028$, which is divisible by 11

12. (d) $a_2 = \frac{a_1 + a_3}{2}$

$a_3 = \frac{a_2 + a_4}{2}$

$a_1 = \frac{a_2 + a_{2012}}{2}$

$a_{2012} = \frac{a_{2001} + a_1}{2}$

Now $a_2 + a_4 + \ldots + a_{2012} = 3018$ (i)

$2a_2 + 2a_4 + \ldots + 2a_{2012} = 6036$

$\therefore\ a_1 + a_2 + a_3 + a_5 + \ldots + a_{2011} + a_1 = 6036$

$\therefore\ 2(a_1 + a_3 + \ldots + a_{2011}) = 6036$

$\therefore\ a_1 + a_3 + \ldots + a_{2011} = 3018$ (ii)

By adding (i) and (i) we get

$a_1 + a_2 + a_3 + \ldots + a_{2012} = 6036$

13. (c) $T_r = \frac{4r+1}{5^r r(r-1)},\ r \geq 2$

$= \frac{5r - (r-1)}{5^r r(r-1)}$

$= \frac{1}{5^{r-1}(r-1)} - \frac{1}{5^r r}$

$\therefore \sum_{r=2}^{\infty} T_r = \left(\left(\frac{1}{5^1 \cdot 1} - \frac{1}{5^2 \cdot 2}\right) + \left(\frac{1}{5^2 \cdot 2} - \frac{1}{5^3 \cdot 3}\right) + \left(\frac{1}{5^3 \cdot 3} - \frac{1}{5^4 \cdot 4}\right) + \ldots \infty\right)$

$= \frac{1}{5}$

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CHAPTER 5

DPP 5.1

Single Correct Answer Type

1. (d)

$$E=\frac{a+4a+4c}{a+c-a}+\frac{4a+4c+c}{a+c-c} \text{ (since } a, b, c \text{ are in A.P.)}$$

$$=5\left(\frac{a}{c}+\frac{c}{a}\right)+8$$

Since a and c are +ve, $\frac{a}{c}+\frac{c}{a}\geq 2$

$\therefore \quad E\geq 10+8$

$\therefore \quad E\geq 18$

2. (a) Using A.M. $\geq$ G.M., we get

$$\frac{\frac{1}{a^4}+\frac{1}{4b^4}}{2}\geq\frac{1}{2} \qquad \text{(as } ab=1\text{)}$$

$$\Rightarrow \quad \frac{1}{a^4}+\frac{1}{4b^4}\geq 1$$

Hence minimum value of $\left(\frac{1}{a^4}+\frac{1}{4b^4}\right)$ is 1.

3. (a) Let $\log_e x=a, \log_e y=b, \log_e z=c$

$$\Rightarrow \quad x=e^a, y=e^b, z=e^c$$

So, given expression $e^{a(b-c)}+e^{b(c-a)}+e^{(a-b)}$

Using A.M. $\geq$ G.M.

$$\therefore \quad \frac{e^{a(b-c)}+e^{b(c-a)}+e^{c(a-b)}}{3}\geq[e^{a(b-c)+b(c-a)+c(a-b)}]^{1/3}$$

$$\therefore \quad e^{a(b-c)}+e^{b(c-a)}+e^{c(a-b)}\geq 3$$

4. (c) Given $27pqr\geq(p+q+r)^3$ (i)

Using A.M. $\geq$ G.M.

$$\Rightarrow \quad \frac{p+q+r}{3}\geq(pqr)^{1/3} \qquad \text{(ii)}$$

$$\Rightarrow \quad p=q=r \qquad \text{[using (i) and (ii)]}$$

Also $3p+4q+5r=12$

$$\Rightarrow \quad p=q=r=1$$

5. (b) $f(x)=\cos^2 x+\frac{\sec x}{4}$

$$=\cos^2 x+\frac{1}{4\cos x}$$

For $x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$, $\cos x>0$

Now $f(x)=\cos^2 x+\frac{1}{8\cos x}+\frac{1}{8\cos x}$

Using A.M. $\geq$ G.M.

$$\Rightarrow \quad \frac{\cos^2 x+\frac{1}{8\cos x}+\frac{1}{8\cos x}}{3}\geq\left(\frac{1}{8^2}\right)^{\frac{1}{3}}=\left[\left(\frac{1}{4}\right)^3\right]^{\frac{1}{3}}$$

$$\Rightarrow \quad \cos^2 x+\frac{1}{4\cos x}\geq\frac{3}{4}$$

6. (b) Using A.M. $\geq$ G.M.

$$\frac{2xy+3yz+4xz}{3}\geq(24x^2y^2z^2)^{\frac{1}{3}}$$

$$\Rightarrow \quad 216\geq 24x^2y^2z^2$$

$$\Rightarrow \quad x^2y^2z^2\leq 9$$

$$\Rightarrow \quad xyz\leq 3$$

So, xyz is greatest when $2xy=3yz=4xz=6$

$$\Rightarrow \quad xy=3, yz=2, zx=3/2$$

$$\Rightarrow \quad \alpha=3/2, \beta=2, \gamma=1$$

$$\Rightarrow \quad 2\alpha+\beta+\gamma=6$$

7. (b) By A.M. $\geq$ G.M.

$x^4+y^4\geq 2x^2y^2$ and $2x^2y^2+z^2\geq\sqrt{8}\,xyz$.

$$\Rightarrow \quad \frac{x^4+y^4+z^2}{xyz}\geq\sqrt{8}$$

8. (d) Using A.M. $\geq$ G.M.

$$\frac{[n_1+(n_2+n_3+n_4)+(n_5+n_6+\cdots+n_9)+\cdots+(\cdots+n_{100})]}{10}\geq\sqrt[10]{k} \text{ (AM} \geq \text{GM)}$$

$$\Rightarrow \quad 2\geq\sqrt[10]{k}$$

$$\Rightarrow \quad 0<k\leq 1024$$

9. (b) $c+a-b, b+c-a, a+b-c$ are all positive

Using A.M. $\geq$ G.M.

$$\therefore \quad \frac{a}{c+a-b}+\frac{b}{a+b-c}+\frac{c}{b+c-a}\geq\left[\frac{abc}{(c+a-b)(a+b-c)(b+c-a)}\right]^{1/3} \qquad \text{(i)}$$

Also, $a^2\geq a^2-(b-c)^2$

$$\Rightarrow \quad a^2\geq(a+b-c)(a-b+c)$$

Similarly $b^2\geq(b+c-a)(b-c+a)$,

$c^2\geq(c+a-b)(c-a+b)$

$$\therefore \quad a^2b^2c^2\geq(a+b-c)^2(b+c-a)^2(c+a-b)^2$$

Thus $abc\geq(a+b-c)(b+c-a)(c+a-b)$

$$\Rightarrow \quad \frac{abc}{(c+a-b)(a+b-c)(b+c-a)}\geq 1$$

Hence, from (i) $\frac{abc}{(c+a-b)(a+b-c)(b+c-a)}\geq 1$

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10. (a) $x+\frac{1}{x}\geq 2,\ldots, x^n+\frac{1}{x^n}\geq 2$

On adding $\left(x+\frac{1}{x}\right)+\left(x^2+\frac{1}{x^2}\right)+\cdots+\left(x^n+\frac{1}{x^n}\right)\geq 2n$

$\therefore \quad \left(\frac{1}{x^n}+\frac{1}{x^{n-1}}+\cdots\frac{1}{x}\right)+1+(x+x^2+\cdots+x^n)\geq 1+2n$

$\therefore \quad \frac{(1+x+\cdots+x^{n-1}+x^n)+x^{n+1}+x^{n+2}+\cdots+x^{2n}}{x^n}\geq 1+2n$

$\therefore \quad \frac{x^n}{1+x+\cdots+2^{2n}}\leq\frac{1}{1+2n}$

DPP 5.2

Single Correct Answer Type

1. (d) b is H.M. of a and c

$$b<\frac{a+c}{2}$$

and $b<\sqrt{ac}$

$\Rightarrow \quad b^2<ac$

$\Rightarrow \quad b-a<c-b$

$\Rightarrow \quad a-b>b-c$

$\Rightarrow \quad \frac{1}{a-b}-\frac{1}{b-c}<0$

2. (c) $\frac{x^2+2xy+2xy+4y^2+z^2+z^2}{6}\geq(16x^4y^4z^4)^{1/6}$

$(x+2y)^2+2z^2\geq 96$

$(x+2y)^2+2z^2-15\geq 81$

$\therefore$ Least value of $\sqrt{(x+2y)^2+2z^2-15}$ is 9

3. (c) $xy=4-(x+y)z$

Now consider $x+y$ and z

Using G.M. $\geq$ A.M., we have

$$\sqrt{(x+y)z}\leq\frac{x+y+z}{2}$$

Also, $(x+y+z)^2$

$=x^2+y^2+z^2+2(xy+yz+zx)$

$=7+8=15$

Now $(x+y)z\leq\frac{(x+y+z)^2}{4}\leq\frac{15}{4}$

$\Rightarrow \quad (x+y)z|_{max}=\frac{15}{4}$

$\Rightarrow \quad xy|_{min}=4-\frac{15}{4}=\frac{1}{4}$

4. (d) Using weighted A.M. and G.M

$$\frac{3\left(\frac{11-x}{3}\right)+5\left(\frac{x+5}{5}\right)}{3+5}\geq\left(\left(\frac{11-x}{3}\right)^3\left(\frac{x+5}{5}\right)^5\right)^{\frac{1}{3+5}}$$

$$\therefore \quad \frac{11-x+x+5}{8}\geq\left(\frac{(11-x)^3(x+5)^5}{3^3\cdot 5^5}\right)^{1/8}$$

$$\therefore \quad (2)^8\geq\frac{(11-x)^3(x+5)^5}{3^3\cdot 5^5}$$

$$\therefore \quad (11-x)^3(x+5)^5\leq 2^8\cdot 3^3\cdot 5^5$$

$$\Rightarrow \quad (11-x)^3(x+5)^5\leq 6^3\cdot 10^5$$

5. (b) $f(x)=(x-1)^2+3(x-1)+1+(x-1)^{-5}$

Consider $x-1=a>0$ (as $x>1$)

$\therefore$ We have $a^2+3a+1+a^{-5}$

Consider quantities $a^2, a, a, a, 1, a^{-5}$

Using A.M. $\geq$ G.M., we have

$$\frac{a^2+a+a+a+1+a^{-5}}{6}\geq 1$$

$\Rightarrow \quad a^2+3a+1+a^{-5}\geq 6$

$\Rightarrow \quad f(x)\geq 6\ \forall\ x>1.$

6. (c) Consider positive numbers

$$\frac{2a}{4},\frac{2a}{4},\frac{2a}{4},\frac{2a}{4},\frac{b}{2},\frac{b}{2},\frac{3c}{2},\frac{3c}{2}$$

Using A.M. $\geq$ G.M.

$$\frac{\frac{2a}{4}+\frac{2a}{4}+\frac{2a}{4}+\frac{2a}{4}+\frac{b}{2}+\frac{b}{2}+\frac{3c}{2}+\frac{3c}{2}}{8}$$

$$\geq\left(\frac{\frac{2a}{4}\cdot\frac{2a}{4}\cdot\frac{2a}{4}\cdot\frac{2a}{4}\cdot\frac{b}{2}\cdot\frac{b}{2}\cdot\frac{3c}{2}\cdot\frac{3c}{2}}{8}\right)^{\frac{1}{8}}$$

$$\Rightarrow \quad \frac{2a+b+3c}{8}\geq\left(\frac{3^2}{2^8}\cdot a^4b^2c^2\right)^{\frac{1}{8}}$$

$$\Rightarrow \quad \frac{1}{8}\geq\frac{(3^2a^4b^2c^2)^{\frac{1}{8}}}{2}$$

$$\Rightarrow \quad a^4b^2c^2\leq\frac{1}{4^8\cdot 3^2}$$

7. (c) Consider the four positive numbers $x, y, \frac{z}{2}, \frac{z}{2}$.

Using A.M $\geq$ G.M., we have

$$\frac{x+y+\frac{z}{2}+\frac{z}{2}}{4}+\geq\left(\frac{xyz^2}{4}\right)^{\frac{1}{4}}$$

$$\Rightarrow \quad xyz^2\leq\frac{(x+y+z)^4}{64}$$

But given that greatest value of xyz^2 is $\frac{1}{64}$

$\Rightarrow \quad x+y+z=1$

But greatest value is attained when $x=y=\frac{z}{2}$

$$\Rightarrow \quad x=\frac{1}{4}, y=\frac{1}{4}, z=\frac{1}{2}$$

8. (a) $x^3-6x^2+3px-2p=0$

$$\text{A.M.}=\frac{x_1+x_2+x_3}{3}=\frac{6}{3}=2$$

$$\text{H.M.}=\frac{3}{\frac{1}{x_1}+\frac{1}{x_2}+\frac{1}{x_3}}=\frac{3x_1x_2x_3}{\Sigma x_1x_2}=2$$

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$\because$ A.M. = H.M. $\Rightarrow x_1 = x_2 = x_3 = 2$

$$\sin^{-1}\left(\frac{1}{x_1}+\frac{1}{x_2}\right)+\cos^{-1}\left(\frac{1}{x_2}+\frac{1}{x_3}\right)-\tan^{-1}\left(\frac{1}{x_3}+\frac{1}{x_1}\right)$$
$$=\frac{\pi}{2}+0-\frac{\pi}{4}=\frac{\pi}{4}$$

Comprehension Type

9. (a) $\dfrac{a^2+b^2}{a+b}+\dfrac{b^2+c^2}{b+c}+\dfrac{a^2+c^2}{a+c}$

$$=\frac{a^2}{a+b}+\frac{b^2}{b+c}+\frac{c^2}{a+c}+\frac{b^2}{a+b}+\frac{c^2}{b+c}+\frac{a^2}{a+c}$$

$$\geq \frac{(2a+2b+2c)^2}{4(a+b+c)} \geq (a+b+c)$$

$$\Rightarrow \quad \frac{a^2}{a+b}+\frac{b^2}{b+c}+\frac{c^2}{a+c}+\frac{b^2}{a+b}+\frac{c^2}{b+c}+\frac{a^2}{a+c} \geq a+b+c$$

10. (b) $\dfrac{1}{a^3(b+c)}+\dfrac{1}{b^3(a+c)}+\dfrac{1}{c^3(a+b)}$

$$=\frac{1/a^2}{ab+ac}+\frac{1/b^2}{ab+bc}+\frac{1/c^2}{ac+bc}$$

$$\geq \frac{\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)^2}{2(ab+bc+ac)} \qquad \text{(i)}$$

Now $\left(\dfrac{1}{a}+\dfrac{1}{b}+\dfrac{1}{c}\right)^2 = \dfrac{(ab+bc+ac)^2}{(abc)^2}$

$$= (ab+bc+ac)^2 \qquad (\text{as } abc = 1)$$

$\therefore$ From (i), $\dfrac{1}{a^3(b+c)}+\dfrac{1}{b^3(a+c)}+\dfrac{1}{c^3(a+b)}$

$$\geq \frac{(ab+bc+ac)}{2}$$

$$\geq \frac{3\cdot\sqrt[3]{(abc)^2}}{2} \qquad \text{(using A.M.} \geq \text{G.M.)}$$

$$\geq \frac{3}{2}$$

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CHAPTER 6

DPP 6.1

Single Correct Answer Type

1. (a) Since one player emerges the winner, each of the remaining 115 players loses in some round. So 115 matches are played.
Alternatively, $58 + 29 + 14 + 7 + 4 + 2 + 1 = 115$

2. (b) First two digits are identical.
$xx_$ → Number of ways $= 9 \times 9 = 81$
$_xx$ → Number of ways $= 9 \times 8 = 72$ (when last two digits are non-zero)
$_00$ → Number of ways = 9 (when last two digits are zero)

3. (d) The number formed by last two digits must be divisible by 4.

3 ways	3 ways	4 ways	4 ways	5 ways	2 ways

(1, 3, 5, 7, 9) (2, 6)

$3 \times 3 \times 4 \times 4 \times 5 \times 2 = 1440$

4. (a) $6^m + 9^n$
Unit digit of 6^m is = 6
Unit digit of 9^n will be = 9 or 1
For multiple of 5 unit digit of 9^n must be = 9
It occur when n = odd
Total number of ordered pair $= 50 \times 25 = 1250$

5. (c) $B_1 B_2 B_3 \ldots B_7 B_8 B_9 B_{10}$
(i) When two terminal books are taken (B_1B_2 or B_9B_{10}) then number of ways $= 2 \times 7 = 14$
(ii) When two consecutive terminal books are not taken (i.e. B_2B_3, B_3B_4, ... B_8B_9)
Then the third book can be selected in 6 ways
Then the number of ways are $7 \times 6 = 42$
∴ Total = 14 + 42 = 56 ways

6. (b) For the first place 5 players (excluding the oldest) and for the remaining places 5(including the oldest) players are available.
∴ No. of ways $= 5 \times 5 \times 4 \times 3 \times 2 \times 1 = 600$

7. (d) Product will be divisible by 3 if atleast one digit is 0, 3, 6, 9
Now total four-digit no. $= 9 \times 10^3$,
Number of four-digit no. without 0, 3, 6, 9 $= 6^4$
Total number of four-digit integers $= 9000 - 1296 = 7704$

8. (a)

1st blank can be filled in 9 ways
2nd blank can be filled in 9 ways (repetition is allowed)
3rd blank can be filled in 9 ways
4th blank can be filled in 9 ways
Now, we have to fill the 5th blank carefully such that the number is divisible by 3.
Add the 4 numbers in the first 4 blanks.
If their sum is in the form $3n$, then fill the last blank by 3, 6 or 9 so that the sum of all digits is divisible by 3.
If their sum is in the form $3n+1$, than fill the last blank by 2, 5 or 8.
If their sum is in the form $3n+2$, than fill the last blank by 1, 4 or 7.
Therefore, in any case, the last blank can be filled in 3 ways only.
∴ Number of five-digit numbers $= 9 \times 9 \times 9 \times 9 \times 3 = 3^9$

9. (a) $770 = 2 \times 5 \times 7 \times 11$
We can assign 2 to x_1, or x_2 or x_3 or x_4.
Thus 2 can be assigned in 4 ways similarly each of 5, 7, 11 can be assigned in 4 ways
No. of solutions $= 4 \times 4 \times 4 \times 4 = 256$

10. (a) Let the square has dimensions $n \times n$.
If n is even, total no. of blue (diagonal) tiles $= n + n = 2n$
If n is odd, then total no. of blue tiles $= n + n - 1 = 2n - 1$
(the middle tile is be counted twice).
Hence, $2n - 1 = 121 \Rightarrow n = 61$
So, number of red tiles $= 61 \times 61 - 121 = 3600$

11. (b) Given $m \le 30$, $n \le 30$
⇒ Total cases $= 30 \times 30 = 900$
[Required condition: $2n \ge m$, $2m \ge n$
Let us find ordered pairs (m, n) such that $2n < m$, $2m < n$. By symmetry we will get same answer for both conditions. Hence, let us evaluate only one $2m < n$

Value of n	No. of points (m, n)
1, 2	0
3, 4	1
5, 6	2
...	
29, 30	14

∴ Required number $= 900 - 2 \times \sum_{r=0}^{14} r = 480$

12. (a) Initially consider the boxes B_1 and B_2 to be distinct.
Object 'a' can be kept in either of the boxes in 2 ways, similarly for all other things ⇒ Total ways $= 2^6$
But this includes when all the things are in B_1 or B_2
⇒ Number of ways $= 2^6 - 2$
Since the boxes are identical, therefore,

$$\frac{2^6 - 2}{2} = 2^5 - 1 = 31$$

13. (b) Least digit used = 4
∴ We can use 4, 5, 6, 7, 8, 9.
So each of the four places can be filled in 6 ways
So number of numbers is 6^4.
But many numbers may not contain 4.
Number of numbers not containing 4 is 5^4 (as each place can now be filled in 5 ways)
So, required number of ways $= 6^4 - 5^4$ (at least one 4)

14. (c) The cases for a_1 {H, T} i.e., $a_1 = 2$
The case for a_2: {HT, TH, TT}, $a_2 = 3$
For $n \ge 3$, if the first outcome is H, then next just T and then a_{n-2}.
If the first outcome is T, then a_{n-1} should follow.
So, $a_n = 1 \times 1 \times a_{n-2} + 1 \times a_{n-1} \Rightarrow a_n = a_{n-2} + a_{n-1}$
So, $a_3 = a_1 + a_2 = 5$, $a_4 = 3 + 5 = 8$ and so on.

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DPP 6.2

Single Correct Answer Type

1. (a) Total no. of arrangement if all the girls do not seat side by side
= [all arrangement – girls seat side by side]
$= 8! - (6! \times 3!)$
$= 6!(56 - 6) = 6! \times 50$
$= 720 \times 50 = 36000$

2. (d) First we have to find all the arrangements of the word 'GENIUS'.
$6! = 720$
Number of arrangements which either start with G or end with S is
$(5! + 5! - 4!) = (120 + 120 - 24) = 216$
Hence, total number of arrangements which neither start with G nor end with S is
$(720 - 216) = 504$

3. (c) Arrange the remaining letters in 5! ways
|P|E|M|T|C
In the 6 gaps, arrange the 3 Ss and 3 Is in
$\frac{6!}{3!3!}$ ways $= 20$
The answer is $5! \times 20 = 2400$

4. (a) The letters other than vowels are SHSHNK which can be arranged in $\frac{6!}{2!2!}$ ways.
Now in its each case, let the first A be placed in the r^{th} gap. Then the number of ways to place the 2nd A will be $(7 - r - 1)$. So,
the total number of ways $= \frac{6!}{2!2!}\sum_{r=1}^{5}(6-r)$
$= \frac{6!}{2!2!} \times (5 + 4 + 3 + 2 + 1)$
$= 2700$

5. (c) The boys can be seated in 5! ways.
If the girls g_1 and g_2 do not want to sit by the side of A_1 (say).
The two gaps $A_6 - A_1$ and $A_1 - A_2$ must be filled by two of the remaining in ${}^4P_2$ ways.
The other four gaps can be filled in 4! ways.
Hence the number of ways $= 5! \times {}^4P_2 \times 4!$
$= 5! \times \frac{4!}{2!} \times 4!$
$= 2.6!4!$

6. (c) Here the sum of the numbers on six cards vanishes.
Case I: If selected 3 cards each of number –1 or 1 i.e
Number of arrangements $= \frac{6!}{3!3!} = 20$
Case II: If selected 2 cards each of no. –1, 0 or 1 i.e
Number of arrangements $= \frac{6!}{2!2!2!} = 90$
Case III: If selected one card each of number –1 and 1 and 4 cards of no. 0.
No. of arrangements $= \frac{6!}{1!1!4!} = 30$
Case IV: If all cards selected from the no. 0
No. of arrangements $= \frac{6!}{6!} = 1$
Hence total no. of arrangements are $20 + 90 + 30 + 1 = 141$

7. (c) The sum of digits must be divisible by 9
The pair of missing numbers must be (1, 2) or (0, 3)
Case I: (1, 2)
⇒ when 1 is first digit the remaining digits (1, 2, 0, 0, 5) can be arranged in 60 days, when 5 or 2 is first digit then in 30 ways each
Case II: (0, 3) will have $3 \times 20 = 60$ ways
So total number of ways $= 60 + 2 \times 30 + 3 \times 20 = 180$

8. (b) 'MEENANSHU'. Number of letters = 9 (EE = 2, NN = 2)
∴ Number of ways = (Total words formed) – (words in which EE together + words in which NN together – words in which EE together and NN together)
$= \frac{9!}{2!\cdot 2!} - \left[\frac{2 \cdot 8!}{2!} - 7!\right]$
$= 18 \cdot 7! - 7 \cdot 7! = (18 - 7)7! = 11 \cdot 7!$

9. (c) A's = 2; B's = 4; C's = 2
Total words formed $= \frac{8!}{4!2!2!} = 420$ (i)
Let ABBC = '×'
Number of ways in which ×ABBC can be arranged $= \frac{5!}{2!} = 60$
But this includes ×ABBC and ABBC×.
The word ABBCABBC is counted twice in 60, hence it should be 59.
Hence, required number of ways $= 420 - 59 = 361$

10. (b) $12 = 2^2 \cdot 3$
Then the four digits can be
1, 1, 6, 2 → no. of numbers $= \frac{4!}{2!} = 12$
1, 1, 3, 4 → no. of numbers $= \frac{4!}{2!} = 12$
2, 2, 3, 1 → no. of numbers $= \frac{4!}{2!} = 12$
Hence, total numbers are 36.

11. (a) The possible arrangements of teachers and students can be as follows:
(i) T S S T S S T S S
(ii) S T S S T S S T S
(iii) S S T S S T S S T
Hence, total number of ways $= 3 \cdot (3!)6! = (18)6!$

DPP 6.3

Single Correct Answer Type

1. (a) The other five letters (other than G, R, L, A, S, N) in 11 places can be arranged in ${}^{11}P_5$ ways.
Then there remains six places.
Choose two of them in ${}^6C_2$ ways. In these places we can put G, R only (i) one way.
Similarly others.
∴ Required no. of ways $= {}^{11}P_5 \times {}^6C_2 \times {}^4C_2 \times {}^2C_2 = 11!/8$

2. (b) The letters other than vowels are: PRMTTN
Number of permutations with no two vowels together is
$\frac{6!}{2!} \times {}^7C_5 \times 5!$
Further among these permutations the number of cases in which T's are together is $5! \times {}^6C_5 \times 5!$

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So the required number

$$= \frac{6!}{2!}{}^{7}C_5 \times 5! - 5! \times {}^{6}C_5 \times 5! = 57 \times (5!)^2$$

3. (c) ${}^{n-2}C_{12-2} = 3^{n-3}C_{12-3}$

$\Rightarrow {}^{n-2}C_{10} = 3^{n-3}C_9$

$\Rightarrow \frac{n-2}{10} = 3 \Rightarrow n = 32$

4. (a) Total selections = ${}^{10}C_3 = 120$

Number of selections in which 3 stations are adjacent = 10

Number of selections in which 2 stations are adjacent = 6

But there are 10 such pairs.

$\Rightarrow$ Total invalid selections = $10 + 6 \times 10 = 70$

5. (a) First arrange 12 persons $A_4, A_5, \ldots A_{15}$ in ${}^{15}P_{12}$ ways.

There remains 3 places. Keep A_1 in the first place and arrange A_2, A_3 in the remaining two places in 2! ways.

$\therefore$ Required no. of arrangements = ${}^{15}P_{12}.1.2!$

6. (a) If Jack picks an apple in ${}^{15}C_1$ ways, then Jill in ${}^{14}C_1{}^{10}C_1$.

If Jack picks a pear in ${}^{10}C_1$ way, then Jill in ${}^{15}C_1{}^{9}C_1$.

$\therefore$ Total no. of ways = ${}^{15}C_1{}^{14}C_1{}^{10}C_1 + {}^{10}C_1{}^{15}C_1{}^{9}C_1$

$= 23 \times 150$

7. (a) Selecting any two pairs, number of ways = ${}^{12}C_2$

Now we have to select remaining three shoes from 10 pairs such that no pair is selected.

Three shoes from different pairs can be selected in ${}^{10}C_3 \times 2^3$ (selecting three pairs and then from each pair selecting on shoe in 2 ways)

$\therefore$ Total number of ways = ${}^{12}C_2 \times {}^{10}C_3 \times 2^3$

8. (c) ${}^{n}C_2 \cdot {}^{n}C_2 = 225 \Rightarrow n = 6$

9. (d) Number of diagonals passing through centre = 6

Number of rectangles = ${}^{6}C_2 = 15$

10. (c) We have $\pi - \frac{2\pi}{n} = \frac{5\pi}{6}$

$\Rightarrow \frac{\pi}{6} = \frac{2\pi}{n}$

$\Rightarrow n = 12$

$\therefore$ Number of diagonals = ${}^{10}C_2 - 12 = 54$

11. (c) First arrange 8 blue alike bottles $\rightarrow$ number of ways = 1

Now select one gap out of 9 gaps created to put two green bottles $\rightarrow$ number of ways = ${}^{9}C_1$

Now select 5 more gaps for other green bottles from remaining 8 gaps $\rightarrow$ number of ways = ${}^{8}C_5$

Hence, total number of ways

$= {}^{9}C_1 \cdot {}^{8}C_5 = 9 \cdot \frac{8 \cdot 7 \cdot 6}{1 \cdot 2 \cdot 3} = 504$.

12. (d) We can take 0 identical and n distinct; 1 identical and $n-1$ distinct; 2 identical and $n-2$ distinct and so on...

$\therefore$ Total no. of solutions = $\sum_{r=0}^{n} {}^{n}C_r = 2^n$

No. of possible subsets of a set containing n elements is 2^n and ${}^{2n+1}C_0 + {}^{2n+1}C_1 + {}^{2n+1}C_2 + \ldots + {}^{2n+1}C_n = 2^{2n}$

13. (c) **Case (i):** zero not taken.

Now we have to select seven digits from 1, 2, 3, 4, 5, 6, 7, 8, 9 so ways are ${}^{9}C_7$

From 7 digit select the largest digits as d and from remaining 6, we can select three digits a, b, c in ${}^{6}C_3$ ways.

Hence number of such numbers are = ${}^{9}C_7 \cdot {}^{6}C_3$

Case (ii): zero taken then

0 must be at last place, then number of such numbers are ${}^{9}C_6 \cdot {}^{5}C_3$

So total ways ${}^{9}C_7 \cdot {}^{6}C_3 + {}^{9}C_6 \cdot {}^{5}C_3 = {}^{9}C_2 \cdot {}^{6}C_3 + {}^{9}C_3 \cdot {}^{5}C_3$

$= 1560$

14. (c) **Case I:** All six digits alike

i.e. 111111, 222222 ... etc. = 5 ways

Case II: 2 alike + 2 other alike + 2 other alike.

Select any three in ${}^{5}C_3$ ways (i.e. 1, 2, 3 and take 11, 22, 33)

For each such selections number of ways = $\frac{6!}{2!2!2!} = 90$

$\Rightarrow$ Total = $10 \times 90 = 900$

Case III: 2 alike + 4 other alike

i.e. 11 2222 or 221111 etc.

Number of ways selecting 2 digits = $({}^{5}C_2)(2) = 20$

For each selections number of ways = $\frac{6!}{2!4!} = 15$

$\Rightarrow$ Total = $20 \times 15 = 300$

Case IV: 3 alike + 3 other alike.

Select any two in ${}^{5}C_2 = 10$ ways

For each selection number of ways = $\frac{6!}{3! \cdot 3!} = 20$

$\Rightarrow$ Total = $10 \times 20 = 200$

Hence total = $5 + 900 + 300 + 200 = 1405$

15. (b) All the possible number are

${}^{9}C_5$ (none containing the digit 0) = 126

Total numbers starting with 1

$= {}^{8}C_4 = 70$ (using 2, 3, 4, 5, 6, 7, 8, 9)

1_ _ _ _

Total numbers starting with 23

$= {}^{6}C_3 = 20$ (using 4, 5, 6, 7, 8, 9)

23 _ _ _

Total numbers starting with 245 = ${}^{4}C_2 = 6$ (using 6, 7, 8, 9)

245 _ _

97th number = 24678

DPP 6.4

Single Correct Answer Type

1. (b) Required number of numbers = ${}^{n}C_0(2)^n + {}^{n}C_2(2)^{n-2} + \ldots$

We know that

$$(1+x)^n + (x-1)^n = 2\sum_{k=0}^{n} {}^{n}C_k x^k$$

Hence, put $x = 2$, get the result

$$2[{}^{n}C_0(2)^n + {}^{n}C_2(2)^{n-2} + {}^{n}C_4(2)^{n-4} + {}^{n}C_6(2)^{n-6} + \ldots]$$

2. (b)

	A	B
C	5	4
D	4	5

Required number of ways of selecting 8 items out of '18' so that selected ones belong equally to A, B, C, D.

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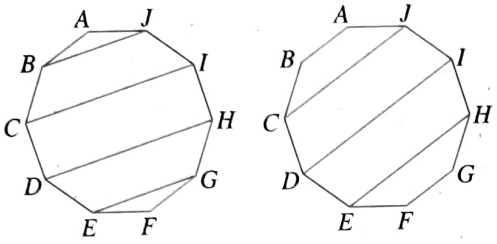
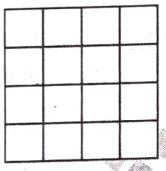
$= ({}^5C_2)^2 \cdot ({}^4C_2)^2 + ({}^5C_1)^2 \cdot ({}^4C_3)^2$
$+ ({}^5C_3)^2 \cdot ({}^4C_1)^2 + ({}^5C_4)^2 + ({}^4C_4)^2$
$= 100 \times 36 + 25 \times 16 + 100 \times 16 + 25 + 1$
$= 5626$

3. (a)



Fix a vertex A.
There are 4 parallel diagonals.
There are 5 such sets since the vertices of the 10-gon can be paired into 5 sets. In each set there are ${}^4C_2$ pairs, i.e, a total of ${}^4C_2 \times 5 = 30$ pairs of diagonals.
Now fix a side AB. There are 3 parallel diagonals and 5 such sets.
So, total pairs $= {}^3C_2 \times 5 = 15$.
Hence, net total $= 30 + 15 = 45$.

4. (c) Selecting two rows for two 'a' in ${}^4C_2$ ways.
Now first 'a' can be put in any row in 4 ways,
Second 'a' can be put in other row in 3 ways as no two a's are in same column.
Thus two 'a' can be filled in
${}^4C_2 \cdot 4 \times 3 = 72$ ways.



Similarly two 'b' can be filled in 72 ways.
Thus two 'a' and two 'b' can be filled in 72^2 ways.
Now, we have to exclude the cases when
(i) both 'a' and 'b' are in same cell i.e., 72 ways (as after fixing 'a' in 72 ways, 'b' can be placed in this position)
(ii) one 'a' and 'b' are in same row or column, i.e., 72×16 ways
Required number of ways $= 72^2 - 72 - 72 \times 16 = 3960$

5. (b) If a_1 is mapped with 2, we have ${}^7C_5$ ways of mapping rest of the elements.
If a_1 is mapped with 3, we have ${}^6C_5$ ways of mapping rest of the elements.
If a_1 is mapped with 4, we have ${}^5C_5$ ways of mapping rest of the elements.
Hence total number of increasing function
$= {}^7C_5 + {}^6C_5 + {}^5C_5 = 28$.

6. (a) $\frac{m}{12} = \frac{12}{n}$
$\Rightarrow \quad mn = 144 \Rightarrow n = \frac{144}{m}$
We want to find here total number of divisors of 144
$144 = 2^4 \cdot 3^2$
Total divisors are $(4 + 1)(2 + 1) = 15$
But negative pairs are also possible hence $= 30$

7. (b) $1000 = 2^3 \cdot 5^3$
Number of divisors $= 16$ which are
$(2^0 + 2^1 + 2^2 + 2^3)(5^0 + 5^1 + 5^2 + 5^3)$
Product of all the 16 divisors is
$= (5^6)(5^6)(5^6)(5^6)(2^6)(2^6)(2^6)(2^6)$
$= 2^{24} \cdot 5^{24}$

8. (b) Selecting atleast one bird of each type $= (2^3 - 1)(2^4 - 1)(2^2 - 1)$

9. (b) $n = 2^\alpha \cdot 3^\beta$
No. of divisors $= (\alpha + 1)(\beta + 1) = 12$ (i)
$2n = 2^{\alpha+1} 3^\beta$
No. of divisors $= (\alpha + 2)(\beta + 1) = 15$ (ii)
$\Rightarrow \quad \frac{\alpha + 2}{\alpha + 1} = \frac{5}{4}$
$\Rightarrow \quad 4\alpha + 8 = 5\alpha + 5 \Rightarrow \alpha = 3$
$\Rightarrow \quad \beta = 2 \Rightarrow 3n = 2^3 3^3$
No. of divisors $= (3 + 1)(3 + 1) = 16$

10. (a) Number of digits are 9
Select 2 places for the digit 1 and 2 in ${}^9C_2$ ways
from the remaining 7 places, select any two places for 3 and 4 in ${}^7C_2$ ways and from the remaining 5 places select any two for 5 and 6 in ${}^5C_2$ ways.
Now, the remaining 3 digits can be filled in 3! ways.
$\therefore$ Total ways $= {}^9C_2 \cdot {}^7C_2 \cdot {}^5C_2 \cdot 3!$

$$= \frac{9!}{2! \cdot 7!} \cdot \frac{7!}{2! \cdot 5!} \cdot \frac{5!}{2! \cdot 3!} \cdot 3!$$

$$= \frac{9!}{8} = \frac{9 \cdot 8 \cdot 7!}{8} = 9 \cdot 7!$$

11. (b) Total number of ways such that at least 3 digits will not come in its position.
$= {}^5C_3 \{3! - {}^3C_1\, 2! + {}^3C_2\, 1! - {}^3C_3\, 0!\}$
$+ {}^5C_4 [4! - {}^4C_1 (3!) + {}^4C_2 (2!) - {}^4C_3 (1!) + {}^4C_4 (0!)\}$
$+ {}^5C_5 \{5! - {}^5C_1\, 4! + {}^5C_2\, 3! - {}^5C_3\, 2! + {}^5C_4\, 1! - {}^5C_5 (0!)\}$
$= 10(2) + 5(9) + (44)$
$= 20 + 45 + 44 = 109$

Multiple Correct Answers Type

12. (a, b, c, d)
(a) ${}^6C_3\, {}^4C_2\, 5!\, 5! = (5!)^3$
(b) ${}^6C_1\, 9!$
(c) $(6 + 1)!\, 4!$
(d) ${}^{10}P_4$

13. (a, b, c, d)
We know that product of r consecutive integer is divisible by $r!$
We have $P = (n - 10)(n - 9)(n - 8) \ldots (n + 10)$ is product of 21 consecutive integers
Which is divisible by 21!
$[(n - 10)(n - 9)(n - 8) \ldots n][(n + 1)(n + 2)(n + 3) \ldots (n + 10)]$
Which is divisible by 11! 10!
$[(n - 10) \ldots (n - 6)] \times [(n - 5) \ldots (n - 1)] \times [n(n + 1) \ldots (n + 4)] \times [(n + 5) \ldots (n + 9)] \times (n + 10)$
Which is divisible by $(5!)^4$.
Also it can be shown that P is divisible by 2! 3! 4! 5! 6!

Comprehension Type

14. (a) The no. of ways of arranging 2's is ${}^{15}C_4$.
Fill the first empty position left after arranging the 2's with a 0 (1 way) and pick the remaining five places from the position of remaining five zeros ${}^{10}C_5$ ways.
$\therefore \quad {}^{15}C_4 \times 1 \times {}^{10}C_5$

15. (b) Put a) in the first position, (1 way). Pick five other positions for the remaining 0's (${}^{14}C_5$ ways), put a 1 in the first of the remaining positions (1 way), then arrange the remaining four 1's (${}^8C_4$ ways)
$\therefore \quad {}^{14}C_5 \times {}^8C_4$

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DPP 6.5

Single Correct Answer Type

1. (a) Required number of ways
= (Number of ways in which 16 players can be divided in 8 couples)
– (Number of ways when S_1 and S_2 are in the same group)

$$= \frac{16!}{2^8 \cdot 8!} - \frac{(14)!}{2^7 \cdot 7!} = \frac{16 \cdot 15 \cdot 14!}{2 \cdot 2^7 \cdot 8 \cdot 7!} - \frac{(14)!}{2^7 \cdot 7!}$$

$$= \frac{(14)!}{2^7 \cdot 7!}\left[\frac{16 \cdot 15}{16} - 1\right] = \frac{(14) \cdot (14)!}{14 \cdot (6!) \cdot 2^6} = \frac{(14)!}{2^6 \cdot 6!}$$

2. (a) Let four variable are x, y, z, w
We have $x^a y^b z^c w^d$ such that $a + b + c + d = 3$
$\therefore$ no. of non-negative integral solutions $= {}^{4+3-1}C_{4-1} = 20$

3. (b) $n(A) = {}^{3+2-1}C_{2-1} \times {}^{3+2-1}C_{2-1} = 16$
$n(B) = n(C) = 16$
$n(A \cap B) = {}^{3+2-1}C_{1-1} \times {}^{3+2-1}C_{1-1}$
$= 1 = n(B \cap C) = n(A \cap C)$
$n(A \cap B \cap C) = 0$
Required no. of ways $= 100 - (16 + 16 + 16 - 1 - 1 - 1 + 0)$
$= 55$

4. (c) Let four different movies are watched by x, y, z and w students.
$\therefore \quad x + y + z + t = 10$
Since each movie is watched by atleast one student,
Number of ways = number of positive integral solution of equation (i)
Hence $= {}^9C_3 = 84$

5. (c) **Case i:** When no box remains empty
Then number of ways of distribution is ${}^{10-1}C_{5-1} = {}^9C_4 = 126$
Case ii: Exactly one box is empty
Number of ways = selection of one box which is empty × distribution of 10 objects in remaining 4 boxes
$= {}^5C_1 \cdot {}^9C_3 = 420$
Case iii: Exactly two remains empty
Number of ways
= selection of two boxes which are empty but not consecutive
× distribution of 10 objects in remaining e boxes
= (selection of any two boxes – two adjacent) ${}^9C_2$
$= ({}^5C_2 - 4) \times {}^9C_2$
$= 6 \times 36 = 216$
Case iv: Exactly three empty.
There is only 1 way to select three empty boxes if no two adjacent
Hence number of ways $1 \cdot {}^9C_1 = 9$
Thus total number of ways $= 126 + 420 + 216 + 9 = 771$

6. (b) Division of objects can be (0, 1, 4) or (0, 2, 3)
So total number of distribution ways

$$= \frac{5!}{0! \cdot 1! \cdot 4!} \times 3! + \frac{5!}{0! \cdot 2! \cdot 3!} \times 3!$$

$= 30 + 60 = 90$ ways

7. (b) $x + y + z = -12$
Here $-10 \le x, y, z \le -1$
Let $x_1 = -x, y_1 = -y, z_1 = -z$
$\therefore$ we have $x_1 + y_1 + z_1 = 12$, where $1 \le x_1, y_1, z_1 \le 12$
$\therefore$ Number of solutions of the equation
$= {}^{12-1}C_{3-1} = {}^{11}C_2 = 55$

8. (b) Five people can be divided into three groups in (1, 1, 3) or (1, 2, 2) ways
Hence, total number of ways

$$= \frac{5!}{3!(1!)^2} \times \frac{1}{2!} + \frac{5!}{(2!)^2 1!} \times \frac{1}{2!}$$

$= 10 + 15 = 25$

9. (b)

Partitioning	Number of ways
4 members	1
1 + 3 members	$\frac{4!}{1!3!} = 4$
2 + 2 members	$\frac{4!}{(2!)^2 2!} = 3$
1 + 1 + 2 members	$\frac{4!}{(1!)^2 2!2!} = 6$
1 + 1 + 1 +1 members	$\frac{4!}{(1!)^4 4!} = 1$
Total	15 ways

10. (b) Number of solutions of the equation $x_1x_2x_3 = y$ is the same as the number of solutions of equation
$x_1x_2x_3x_4 = 30 = 2 \times 3 \times 5$;
where x_4 is dummy variable
Now number of solutions = number of ways distinct integers 2, 3 and 5 can be distributed in four boxes x_1, x_2, x_3 and $x_4 = 4^3 = 64$

Comprehension Type

11. (b) For 1st and event there are 7 ways
2nd event there are 7 ways
8th event there are 7 ways
$\Rightarrow$ Total no of ways $= 7^8$.

12. (a) $1 + 1 + 1 + 1 + 1 + 1 + 2 = 8$

$$\text{Total number of ways} = \frac{8!}{2!(1!)^6 6!} \times 7! = 28 \times 5040$$

13. (b) $1 + 1 + 1 + 1 + 1 + 3 = 8$
$1 + 1 + 1 + 1 + 2 + 2 = 8$

$$\text{Total number of ways} = \left[\frac{8!}{3!(1!)^5 5!} + \frac{8!}{(1!)^4 (2!)^2 2!4!}\right] \times 7!$$

$= 266 \times 7!$

14. (d) For every choice of $r = 1, 2, 3, \ldots (n-1)$ when the n^{th} object a_n goes to the rth place, there are $D_{n-1} + D_{n-2}$ ways of the other $(n-1)$ objects $a_1, a_2, \ldots, a_{n-1}$ to be deranged.
Hence $D_n = (n-1)(D_{n-1} + D_{n-2})$

15. (a) $D_n - nD_{n-1} = (-1)(D_{n-1} - (n-1)D_{n-2})$
By implied induction on n, we obtain
$D_n - nD_{n-1} = (-1)^{n-2}(D_2 - 2D_1)$, where $D_1 = 0$ and $D_2 = 1$
$= (-1)^n$

16. (b) $\frac{D_n}{n!} - \frac{D_{n-1}}{(n-1)!} = \frac{(-1)^n}{n!}$ gives

$$\frac{D_n}{n!} = \sum_{r=2}^{n}\left(\frac{D_r}{r!} - \frac{D_{r-1}}{(r-1)!}\right) = \sum_{r=2}^{n}\frac{(-1)^r}{r!}$$

$$\Rightarrow \quad D_n = n!\sum_{r=2}^{n}\frac{(-1)^r}{r!}$$

$$\therefore \quad D_5 = 5!\left(\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!}\right) = 44$$

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CHAPTER 7

DPP 7.1

Single Correct Answer Type

1. (b) The coefficient of x^6 in the given expression = coefficient of x^6 in

$(1+{}^6C_1x^6)(1+{}^5C_1x^5)(1+{}^4C_1x^4)(1+{}^3C_1x^3+{}^3C_2x^6)(1+{}^2C_1x^2+{}^2C_2x^4)(1+x)$

= coefficient of x^6 in $(1+6x^6+5x^5+4x^4)(1+2x^2+3x^3+x^4+6x^5+3x^6)(1+x)$

= coefficient of x^6 in $(11x^5+17x^6)(1+x) = 28$

2. (b) General term of $\left(x-\frac{3}{x}\right)^{11}$

$$T_{r+1} = {}^{11}C_r\, x^{11-r}\left(-\frac{3}{x}\right)^r = {}^{11}C_r\, x^{11-2r}(-3)^r$$

$11-2r \neq 0, -2$ so $11-2r=-1$ is only possible case

$\therefore \quad r=6$

$\therefore$ coefficient is $(-3)^6\,{}^{11}C_6$

3. (d) 13^{th} term in expansion $(x^2+2/x)^n$ is

$${}^nC_{12}(x^2)^{n-12}(2/x)^{12} = {}^nC_{12}x^{2n-36}$$

Given 13^{th} term is independent of x

$\therefore \quad n=18$

$\therefore$ Divisors of 18 are 1, 2, 3, 6, 9, 12, 18 sum of which is 51

4. (a) $(1+x+x^2+x^3+x^4)^{1001}(1-x)^{1002}$

$\left(\frac{1-x^5}{1-x}\right)^{1001}(1-x)^{1002} = (1-x)(1-x^5)^{1001}$, so all the powers of x will be of the $5m$ or $5m+1$ $(m \in I)$

So coefficient of x^{2009} will be 0

5. (a) For $\left(x^2-\frac{1}{x}\right)^n$

$$T_{r+1} = {}^nC_r\,(x^2)^{n-r}(-1)^r x^{-r} = {}^nC_r\,x^{2n-3r}(-1)^r$$

Constant term $= {}^nC_r(-1)^r$ if $2n=3r$ i.e. coefficient of $x=0$

Hence ${}^nC_{2n/3}(-1)^{2n/3} = 15 = {}^6C_4 \Rightarrow n=6$

6. (b) $\left(px^{\frac{1}{12}}+qx^{-\frac{1}{9}}\right)^{14}$

General term $T_{r+1} = 14C_r\left(px^{\frac{1}{12}}\right)^{14-r}\left(qx^{-\frac{1}{9}}\right)^r$

$$= {}^{14}C_r\,p^{14-r}q^r\,x^{\frac{14-r}{12}-\frac{r}{9}}$$

Term is independent of r, then $\frac{14-r}{12}-\frac{r}{9}=0$

$\therefore r=6$

$\therefore$ Term independent of x is ${}^{14}C_5\,p^8\,q^6 = {}^{14}C_6\,(p^4\,q^3)^2$

Now p^4, q^3 are positive

Using AM $\geq$ GM

$$\frac{p^4+q^3}{2} \geq (p^4q^3)^{1/2} \Rightarrow (p^4q^3)^2 \leq 1$$

$\Rightarrow$ Maximum value of term independent of x is ${}^{14}C_6$.

7. (c) $\left(\frac{1}{x^2}-x^3\right)^n$

$$T_{r+1} = \frac{n!}{r!(n-r)!}(-1)^{n-r}x^{5r-2n}$$

If $5r-2n=5$, then $5r=2n+5$

$$r = \frac{2n}{5}+1$$

If $5r-2n=10$, then $5r=2n+10$

$$r = \frac{2n}{5}+2$$

Let $n=5k$

According to question

$$\frac{5k!}{(2k+1)!(3k-1)!}-\frac{5k!}{(2k+2)!(3k-2)!}=0$$

$$\frac{1}{3k-1}-\frac{1}{2k+2}=0$$

$k=3 \quad n=15$

8. (c) $((1+2t^2)-t^3)^9$

$= {}^9C_0(1+2t^2)^9 - {}^9C_1(1+2t^2)^8\cdot t^3 + {}^9C_2(1+2t^2)^7\cdot t^6 - {}^9C_3(1+2t^2)^6\cdot t^9 + \ldots - {}^9C_9(t^3)^9$

$\therefore$ Coefficient of t^8 in the expansion of $(1+2t^2-t^3)^9$

$= {}^9C_0$(coefficient of t^8 in $(1+2t^2)^9$)

$- {}^9C_1$(coefficient of t^5 in $(1+2t^2)^8$)

$+ {}^9C_2$(coefficient of t^2 in $(1+2t^2)^7$)

$= {}^9C_0\cdot{}^9C_4\,2^4 - 0 + {}^9C_2\cdot{}^7C_1\cdot 2$

$= 2520$

9. (d) $T_{r+1} = {}^{18}C_r\cdot(9x)^{18-r}\cdot(-1)^r\cdot\frac{(x)^{-r/2}}{3^r}$

$$T_{r+1} = {}^{18}C_r\cdot 9^{18-r}\cdot\frac{(-1)^r}{3^r}\cdot x^{18-\frac{3r}{2}}$$

$\therefore \quad 18-\frac{3r}{2}=0$

$\Rightarrow \quad r=12$

According to question ${}^{18}C_{12}\cdot\frac{9^6}{3^{12}} = \alpha\cdot{}^{18}C_2$

$\Rightarrow \quad \alpha=1$

10. (c) General term $T_{r+1} = {}^{16}C_r\left(\frac{x}{\cos\theta}\right)^{16-r}\left(\frac{1}{x\sin\theta}\right)^r$

$$= {}^{16}C_r\frac{1}{(\cos\theta)^{16-r}(\sin\theta)^r}\cdot x^{16-2r}$$

If this term is independent of x, then $16-2r=0$

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$\therefore$ The term independent of $x = {}^{16}C_8 \dfrac{1}{\cos^8\theta \sin^8\theta}$

$= {}^{16}C_8 \dfrac{2^8}{\sin^8 2\theta}$

$I_1 = {}^{16}C_8 \dfrac{2^8}{\sin^8 \frac{\pi}{2}} = {}^{16}C_8\, 2^8$

$I_2 = {}^{16}C_8 \dfrac{2^8}{\sin^8 \frac{\pi}{4}} = {}^{16}C_8 \cdot \dfrac{2^8}{\left(\frac{1}{\sqrt{2}}\right)^8} = {}^{16}C_8\, 2^{12}$

$\therefore \quad \dfrac{I_2}{I_1} = \dfrac{2^{12}}{2^8} = 2^4 = 16$

1. (b) Clearly, $a_{i,j} = \dfrac{2010!}{i!\, j!(2010-i-j)!}$

and $a_{j,i} = \dfrac{2010!}{j!i!(2010-i-j)!}$

Hence, $a_{i,j} = a_{j,i}$

2. (a) The given series

$$S = (1+x)^{500}\left[1 + \frac{x}{1+x} + \left(\frac{x}{1+x}\right)^2 + \ldots + \left(\frac{x}{1+x}\right)^{500}\right]$$

$$= (1+x)^{500} \times \frac{1-\left(\frac{x}{1+x}\right)^{501}}{1-\frac{x}{1+x}}$$

$$= (1+x)^{501} - x^{501}$$

Hence, the coefficient of x^{301} in $S = {}^{501}C_{301}$.

3. (a) The highest power of $x = 1 + 2 + 3 + \ldots + 12 = 78$

to get coefficient of x^{70}, we have to omit the factors containing x^8

(1) Product of the constant terms of $(x-1)(x^7-7) = 7$
(2) Product of the constant terms of $(x^2-2)(x^6-6) = 12$
(3) Product of the constant terms of $(x^3-3)(x^5-5) = 15$
(4) Product of the constant terms of $(x-1)(x^2-2)(x^5-5) = -10$
(5) Product of the constant terms of $(x-1)(x^3-3)(x^4-4) = -12$

Required coefficient $= 7 + 12 + 15 - 10 - 12 - 8$
$= 34 - 30 = 4$

4. (b) Given $\{(1-x)(1+x+x^2)\}^n$

$$= \sum_{k=0}^{n} a_k x^k \frac{(1-x)^{3n}}{(1-x)^{2k}}$$

$$\left\{\frac{1+x+x^2}{(1-x)^2}\right\} = \sum_{k=0}^{n} a_k \frac{x^k}{(1-x)^{2k}}$$

$$\left\{\frac{(1-x)^2+3x}{(1-x)^2}\right\}^n = \sum_{k=0}^{n} a_k \left\{\frac{x}{(1-x)^2}\right\}^k$$

$$\left\{1+\frac{3x}{(1-x)^2}\right\}^n = \sum_{k=0}^{n} a_k \left\{\frac{x}{(1-x)^2}\right\}^k$$

Put $\dfrac{x}{(1-x)^2} = t$

$$(1+3t)^n = \sum_{k=0}^{n} a_k t^k$$

$3 \cdot a_{k-1} + a_k = 3$(coefficient of t^{k-1}) + coefficient of t^k

$= 3.3^{k-1}\,{}^nC_{k-1} + 3^k\,{}^nC_k$

$= 3^k({}^nC_{k-1} + {}^nC_k)$

$= 3^k({}^{n+1}C_k)$

DPP 7.2

Single Correct Answer Type

1. (d) $x^{2001} - \left[{}^{2001}C_0 x^{2001} - {}^{2001}C_1 x^{2000} \cdot \dfrac{1}{2} + {}^{2001}C_2 x^{1999} \cdot \dfrac{1}{4} \ldots\right] = 0$

$\therefore \quad {}^{2001}C_1 x^{2000} \cdot \dfrac{1}{2} - {}^{2001}C_2 x^{1999} \cdot \dfrac{1}{4} \ldots = 0$

sum of the roots $= {}^{2001}C_2 \cdot \dfrac{1}{4} \cdot \dfrac{2}{{}^{2001}C_1} = \dfrac{2001 \cdot 2000}{4 \cdot 2001} = 500$

2. (d) Given expression is

$$\left\{\sqrt{x^{\frac{1}{1+\log_{10} x}}} + \sqrt[12]{x}\right\}^6 = \left\{x^{\frac{1}{2}\left(\frac{1}{1+\log_{10} x}\right)} + x^{\frac{1}{12}}\right\}^6$$

$T_4 = 200$

$$\Rightarrow \quad {}^6C_3 \cdot x^{\frac{1}{2}\left(\frac{1}{1+\log_{10} x}\right)^{6-3}} \times \left(x^{\frac{1}{12}}\right)^3 = 200$$

$$\Rightarrow \quad x^{\frac{3}{2}\frac{1}{1+\log_{10} x} + \frac{1}{4}} = 10$$

$$\Rightarrow \quad \frac{3}{2(1+\log_{10} x)} + \frac{1}{4} = \log_x 10$$

$$\Rightarrow \quad \frac{3}{2(1+\log_{10} x)} + \frac{1}{4} = \frac{1}{\log_{10} x}$$

Put $\log_{10} x = y$

$$\Rightarrow \quad \frac{3}{2(1+y)} + \frac{1}{4} = \frac{1}{y}$$

$$\Rightarrow \quad y^2 + 3y - 4 = 0$$

$$\Rightarrow \quad y = -4,\ y = 1$$

$$\Rightarrow \quad \log_{10} x = -4 \qquad \log_{10} x = 1$$

$$\Rightarrow \quad x = 10 \text{ (as } x > 1)$$

3. (c) To get common terms in both the expansions

$$x^n (y^2)^{13-n} = (y)^{14-n} (x^2)^n$$

$\therefore \quad r_1 = 2r_2$ and $26 - 2r_1 = 14 - r_2$

$\therefore \quad r_1 = 8,\ r_2 = 4$

$\therefore$ Only one term is common.

4. (d) $\displaystyle\sum_{r=1}^{n} {}^nC_r((1+2)^r - 2^r)$

$$= \sum_{r=1}^{n} {}^nC_r \cdot 3^r - \sum_{r=1}^{n} {}^nC_r \cdot 2^r$$

$$= (4^n - 1) - (3^n - 1) = 4^n - 3^n$$

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5. (a) Let the term is $T_{r+1} = {}^nC_r (x^3)^{n-r}\left(\frac{-2}{\sqrt{x}}\right)^r$

$\therefore 3n - 3r - r/2 = 2 \Rightarrow n = \frac{4+7r}{6}$

$\therefore$ Least value of n which is double digit = 10

6. (b) $T_{r+1} = {}^nC_r\, a^{n-r}\,.\,b^r$ where $a = 2^{1/3}$ and $b = 3^{-1/3}$

T_7 from beginning $= {}^nC_6\, a^{n-6}\, b^6$ and T_7 from end $= {}^nC_6 b^{n-6}\, a^6$

$$\Rightarrow \frac{a^{n-12}}{b^{n-12}} = \frac{1}{6}$$

$$\Rightarrow 2^{\frac{n-12}{3}} \cdot 3^{\frac{n-12}{3}} = 6^{-1}$$

$$\Rightarrow n - 12 = -3 \Rightarrow n = 9$$

7. (c) $\left(1 + x^3 + \frac{1}{x^3}\right)^{200}$

$$= 1 + {}^{200}C_1\left(x^3 + \frac{1}{x^3}\right) + {}^{200}C_2\left(x^3 + \frac{1}{x^3}\right)^2 \cdots {}^{200}C_{200}\left(x^3 + \frac{1}{x^3}\right)^{200}$$

The RHS is of the form

$$k_0 + k_1 x^3 + k_2 (x^3)^2 + \ldots.. k_{200} (x^3)^{200} + \frac{l_1}{x^3} + \frac{l_2}{(x^3)^2} + \ldots.. \frac{l_{200}}{(x^3)^{200}}$$

where $k_0, k_1, \ldots, l_1, l_2, \ldots\ldots$ are all real constants.

$\therefore$ Number of terms = 1 + 200 + 200 = 401

8. (c) $T_r = {}^nC_{r-1} p^{n-r+1} \cdot q^{r-1}$

$= T_{r+1} = {}^nC_r p^{n-r} \cdot q^r$ (given)

$$\Rightarrow p^{n-r+1} \cdot q^{r-1} = \frac{n-r+1}{r} p^{n-r} \cdot q^r$$

$$\Rightarrow \frac{p}{(n-r+1)} = \frac{q}{r}$$

$$\Rightarrow \frac{(n+1)q}{(p+q)r} = 1$$

9. (c) $(3 + a\sqrt{2})^{100} + (3 + b\sqrt{2})^{100} = 7 + 5\sqrt{2}$

since a and b are rational

$(3 - a\sqrt{2})^{100} + (3 - b\sqrt{2})^{100} = 7 - 5\sqrt{2}$

Here R.H.S. < 0 but L.H.S. > 0

So there are no such pairs (a, b)

10. (c) $(1 - 3x + 3x^2 - x^3)^{2n} = (1 - x)^{6n}$

middle term is $\left(\frac{6n}{2} + 1\right)^{\text{th}}$

i.e., $(3n + 1)^{\text{th}}$ term is middle term,

$$T_{3n+1} = {}^{6n}C_{3n}(-x)^{3n} = \frac{6n!}{3n!3n!}(-x)^{3n}$$

11. (b) $(3 - 2x)^{15}$ at $x = \frac{4}{3}$

For greatest term, $r \le \frac{16|2x|}{3 + |2x|} = \frac{16 \times 2 \times \frac{4}{3}}{3 + \frac{8}{3}} = \frac{128}{17}$

$\therefore \quad r = 7$

$\therefore \quad t_{r+1} = t_8 = {}^{15}C_7 3^8 (-2x)^7 = -{}^{15}C_7 3^8 \left(\frac{8}{3}\right)^7$ is numerically greatest term.

$\therefore\ t_7$ and t_9 are positive terms

$$t_7 = {}^{13}C_6\, 3^9\, (-2x)^6 = {}^{15}C_6\, 3^9 (8/3)^6 = {}^{15}C_6\, 8^6 \cdot 3^3$$

and $t_9 = {}^{15}C_8\, 3^7\, (-2x)^8 = {}^{15}C_8 \cdot 3^7 (8/3)^8 = {}^{15}C_8\, 8^8 \cdot 3^{-1}$

$$\therefore \quad t_9 - t_7 = \frac{15!}{8!7!}\frac{8^8}{3} - \frac{15!}{8!6!}8^6.3^3 = \frac{15!}{9!6!}\frac{8^6}{3}\left[\frac{64}{7} - \frac{3^4}{9}\right] > 0$$

$\therefore \quad t_7 < t_9$

$$t_{11} = {}^{15}C_{10}\frac{8^{10}}{3^5}$$

$\therefore \quad t_7 - t_{11} > 0$

$\therefore \quad t_7 > t_{11}$

Thus $t_9 > t_7 > t_{11}$

Hence t_7 is the second largest term.

12. (c) For T_6 to be numerically greatest term

$$5 < \frac{n+1}{1 + \left|\frac{3/2}{3/3}\right|} < 6$$

$$5 < \frac{2}{5}(n+1) < 6$$

$$\frac{25}{2} < n + 1 < 15$$

$$\frac{23}{2} < n < 14$$

$\therefore \quad n = 12, 13$

13. (b) $f^2 - f + pf - p$

$= -f(1 - f) - p(1 - f)$

$= -[(p + f)(1 - f)]$

where $p + f = (5 + 2\sqrt{6})^n$

and $1 - f = f' = (5 - 2\sqrt{6})^n$

hence $f^2 - f + pf - p = -1$

14. (a) We have

$$3^{100} = (3^4)^{25}$$

$$= (81)^{25}$$

$$= (80 + 1)^{25}$$

$$= {}^{25}C_0 \cdot (80)^{25} + {}^{25}C_1 \cdot (80)^{24} + \ldots + {}^{25}C_{22}(80)^3 + {}^{25}C_{23}(80)^2 + {}^{25}C_{24}(80) + {}^{25}C_{25}$$

$$= 10^3[{}^{25}C_0\, 8^{25} \times 10^{22} + {}^{25}C_1 \times 8^{24} \times 10^{21} + \ldots + {}^{25}C_{22} \times 8^3] + \frac{25 \times 24}{2} \times (80)^2 + 25 \times 80 + 1$$

$$= 10^3 m + 1920000 + 2000 + 1, \text{ where } m \in N$$

Thus, last three digits of 3^{100} are 001,

15. (d) $(27)^{10} + 7^{51} = (30 - 3)^{10} + (10 - 3)^{51}$

$$= 3^{10} - 3^{51} + 10\lambda$$

$$= (10 - 1)^5 - 3(10 - 1)^{25} + 10\lambda$$

$$= -1 + 3 + 10\lambda_1$$

$$= 2 + 10\lambda_1$$

$\therefore$ Remainder is 2

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(b) Let $T_{r+1} \geq T_r$

$\Rightarrow \frac{{}^nC_r}{(r+1)(r+2)(r+3)} \geq \frac{{}^nC_{r-1}}{(r)(r+1)(r+2)}$

$\Rightarrow \frac{{}^nC_r}{{}^nC_{r-1}} \geq \frac{r+3}{r}$

$\Rightarrow \frac{n-r+1}{r} \geq \frac{r+3}{r}$

$\Rightarrow r \leq \left(\frac{n-2}{2}\right)$

For $n = 50 \Rightarrow r \geq 24$

So 24th and 25th terms of sequence are greatest.

DPP 7.3

Single Correct Answer Type

(b) $P_n = {}^nC_0\,{}^nC_1\,{}^nC_2 \ldots {}^nC_n$

$\frac{P_{n+1}}{P_n} = \frac{{}^{n+1}C_1}{{}^nC_1} \cdot \frac{{}^{n+1}C_2}{{}^nC_2} \cdots \frac{{}^{n+1}C_n}{{}^nC_n} \cdot {}^{n+1}C_{n+1}$

$= \frac{n+1}{n} \cdot \frac{n+1}{n-1} \cdot \frac{n+1}{n-2} \cdots \frac{n+1}{1} = \frac{(n+1)^n}{n!}$

$\Rightarrow \frac{(n+1)^n}{n!} = \frac{10^9}{9!}$

$\Rightarrow n = 9$

(a) $S = {}^{39}P_{19} + {}^{38}P_{19} + {}^{37}P_{19} + \ldots + {}^{20}P_{19}$

$= 19!\,\{{}^{30}C_{19} + {}^{38}C_{19} + {}^{37}C_{19} + \ldots + {}^{20}C_{19}\}$

$= 19!\,[{}^{39}C_{19} + {}^{38}C_{19} + {}^{37}C_{19} + \ldots + ({}^{20}C_{19} + {}^{20}C_{10}) - 1]$

$= 19!\,[{}^{39}C_{19} + {}^{38}C_{19} + \ldots + ({}^{21}C_{19} + {}^{21}C_{20}) - 1]$

$= 19!\,\{{}^{40}C_{20} - 1\}$

$= \frac{(41-1)(41-2)(41-3)\ldots.(41-20)-20!}{20} = \frac{41k}{20}$

$\Rightarrow$ greatest prime number is 41.

(c)

Let $t_r = \frac{{}^nC_{r-1}}{{}^nC_r + {}^nC_{r-1}} = \frac{1}{\frac{{}^nC_r}{{}^nC_{r-1}} + 1} = \frac{1}{\frac{n-r+1}{r} + 1}$

$\therefore \quad t_r = \frac{r}{n+1}$

Now,

$S = \sum_{r=0}^{n} (t_r)^3$

$\Rightarrow S = \sum_{r=0}^{n} \frac{r^3}{(n+1)^3} = \frac{1}{(n+1)^3} \sum_{r=0}^{n} r^3$

$\Rightarrow S = \frac{1}{(n+1)^3}\left\{\frac{n(n+1)}{2}\right\}^2 \Rightarrow S = \frac{n^2}{4(n+1)}$

Now, $S = \frac{25}{24}$ (given) $\quad \therefore n = 5.$

4. (a) $a = {}^nC_{r-1}, b = {}^nC_r, c = {}^nC_{r+1}, d = {}^nC_{r+2}$

$a + b = {}^{n+1}C_r$

$b + c = {}^{n+1}C_{r+1}$

$c + d = {}^{n+1}C_{r+2}$

$\frac{a+b}{a} = \frac{n+1}{r}$

$\Rightarrow \frac{a}{a+b} = \frac{r}{n+1}, \frac{b}{b+c} = \frac{r+1}{n+1}, \frac{c}{c+d} = \frac{r+2}{n+1}$

$\therefore \quad \frac{a}{a+b}, \frac{b}{b+c}, \frac{c}{c+d}$ are in A.P.

A.M. > G.M.

$\frac{b}{b+c} > \sqrt{\frac{ac}{(a+b)(c+d)}}$

$\Rightarrow \left(\frac{b}{b+c}\right)^2 - \frac{ac}{(a+b)(c+d)} > 0$

5. (c)

Consider,

$(\cos\theta - i\sin\theta)^m$

$= {}^mC_0\cos^m\theta - {}^mC_1\cos^{m-1}\theta i\sin\theta + \ldots + {}^mC_m(-i\sin\theta)^m$ (i)

$(\cos\theta + i\sin\theta)^m$

$= {}^mC_0\cos^m\theta + {}^mC_1\cos^{m-1}\theta i\sin\theta + \ldots + {}^mC_m(i\sin\theta)^m$ (ii)

Adding (1) and (2), we get

$2\cos m\theta = 2[{}^mC_0\cos^m\theta - {}^mC_2\cos^{m-2}\theta\sin^2\theta \ldots]$ (iii)

Subtracting (1) from (2), we get

$2i\sin m\theta = 2i[{}^mC_1\cos^{m-1}\theta\sin\theta - {}^mC_3\cos^{m-3}\theta\sin^3\theta \ldots]$ (iv)

Adding (3) and (4), we get

$\cos m\theta + \sin m\theta$

$= [{}^mC_0\cos^m\theta + {}^mC_1\cos^{m-1}\theta\sin\theta - {}^mC_2\cos^{m-2}\theta\sin^2\theta - {}^mC_3\cos^{m-3}\theta\sin^3\theta \ldots]$

$\Rightarrow \sqrt{2}\sin\left(m\theta + \frac{\pi}{4}\right)$

$= [{}^mC_0\cos^m\theta + {}^mC_1\cos^{m-1}\theta\sin\theta - {}^mC_2\cos^{m-2}\theta\sin^2\theta - {}^mC_3\cos^{m-3}\theta\sin^3\theta \ldots]$

Putting $\theta = \frac{\pi}{4}$, we get

$\sqrt{2}\sin\left(\frac{(m+1)\pi}{4}\right) = \frac{1}{2^{m/2}}$

$[({}^mC_0 + {}^mC_1 - {}^mC_2 - {}^mC_3) + ({}^mC_4 + {}^mC_5 - {}^mC_6 - {}^mC_7) + \ldots + ({}^mC_{m-3} + {}^mC_{m-2} - {}^mC_{m-1} - {}^mC_m)]$

Hence, $m + 1 = 4k$, for given quantity to be 0.

$\Rightarrow m = 4k - 1$, where $k \in N$

6. (d) $\sum_{r=0}^{3n-1} (-1)^r\,{}^{6n}C_{2r+1}3^r$

$=$ Imaginary part of $\sum_{r=0}^{3n-1} \frac{1}{\sqrt{3}i} \cdot {}^{6n}C_{2r+1}(\sqrt{3}i)^{2r+1}$

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$$= \frac{1}{i\sqrt{3}} \text{Im}(1+i\sqrt{3})^{6n}$$

$$= \frac{1}{i\sqrt{3}} \text{Im}(2^6(\cos 2\pi + i \sin 2\pi))$$

$$= 0$$

7. (a) $(x + {}^{101}C_0)(x + {}^{101}C_0) \ldots (x + {}^{101}C_{50})$ contains 51 linear factors.

$\therefore x^{50}$ is obtained by multiplying x's from any 50 factors and the constant term from the remaining one factor.

Hence the coefficient of $x^{50} = S = {}^{101}C_0 + {}^{101}C_1 + \ldots + {}^{101}C_{50}$

${}^{101}C_{51} = {}^{101}C_{50}$, ${}^{101}C_{101} = {}^{101}C_0$ etc.

$\therefore 2S = {}^{101}C_0 + {}^{101}C_1 + \ldots + {}^{101}C_{50} + {}^{101}C_{51} + \ldots + {}^{101}C_{101} = 2^{101}$

$\Rightarrow \quad S = 2^{100} \Rightarrow 4^{50}$

8. (b) The sum of the coefficients of odd powers in the expansion of $(1 + x)^n$

= sum of the coefficients of even powers in $(1 + x)^n$

$= 2^{n-1} = 2^{70-1} = 2^{69}$

9. (c) The sum of coefficients of terms not containing $y = 3^6$ (putting $x = z = w = 1$ and $y = 0$)

The sum of coefficients of terms not containing both x and $y = 2^6$ (putting $z = w = 1$ and $x = y = 0$)

So the required number $= 3^6 - 2^6$

10. (a) Since ${}^{10}C_0 + {}^{11}C_1 + {}^{12}C_2 + {}^{13}C_3 + \ldots + {}^{999}C_{989}$

$= {}^{1000}C_{989} = {}^{1000}C_{11}$

(Since, ${}^{10}C_0 = {}^{11}C_0$ and ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$)

So, ${}^{12}C_2 + {}^{13}C_3 + {}^{14}C_4 + \ldots + {}^{999}C_{989} = {}^{1000}C_{11} - 12$

11. (a) Putting $x = 1$ and -1 and adding

$$a_0 + a_2 + \ldots + a_{50} = \frac{3^{25}+1}{2}$$

$$= \frac{(1+2)^{25}+1}{2}$$

$$= \frac{{}^{25}C_0 + {}^{25}C_1 \cdot 2 + {}^{25}C_2 \cdot 2^2 + {}^{25}C_{25} \cdot 2^{25} + 1}{2}$$

$$= \frac{2[1 + {}^{25}C_1 + {}^{25}C_2 \cdot 2 + \ldots + {}^{25}C_{25} \cdot 2^{24}]}{2}$$

$$= 2[13 + {}^{25}C_2 + \ldots + {}^{25}C_{25} \cdot 2^{23}]$$

12. (b) Sum of coefficient of $(q + r)^{20}(1 + (p - 2)x)^{20}$

$= (q + r)^{20}(p - 1)^{20}$ [put $x = 1$]

Square of the sum of coefficient of $(2rqx - (r + q).y)^{10}$

$= (2rq - (r + q))^{20}$ [put $x = y = 1$]

So $(q + r)^{20}(p - 1)^{20} = (2rq - (q + r))^{20}$

$$\Rightarrow \quad p - 1 = \frac{2rq}{r+q} - 1$$

$$\Rightarrow \quad p = \frac{2rq}{r+q}$$

$\Rightarrow$ p = H.M. of r and q

$\leq$ A.M. of r and q

$= (r + q)/2$

13. (b) $S_n = {}^{3n}C_0 - {}^{3n}C_1 + {}^{3n}C_2 + \ldots + (-1)^n \cdot {}^{3n}C_n$

But ${}^{3n}C_0 = {}^{3n-1}C_0$

$- {}^{3n}C_1 = - {}^{3n-1}C_0 - {}^{3n-1}C_1$

$- {}^{3n}C_2 = {}^{3n-1}C_1 + {}^{3n-1}C_2$

$- {}^{3n}C_3 = - {}^{3n-1}C_2 - {}^{3n-1}C_3$

.....

$(-1)^n \cdot {}^{3n}C_n = (-1)^n \cdot {}^{3n-1}C_{n-1} + (-1)^n \cdot {}^{3n-1}C_n$

On adding we get $S_n = (-1)^n \cdot {}^{3n-1}C_n$

14. (d) Coefficient of $x^k = a_k = {}^{n+1}C_{k+1}$

$$a_{n-2} = {}^{n+1}C_{n+1} = \frac{n(n+1)}{2}$$

$$a_9^2 - a_8^2 = {}^{n+2}C_{10}({}^{n+1}C_{10} - {}^{n+1}C_9)$$

$$a_p > a_{p-1} \Rightarrow p < \frac{n}{2}$$

$$\sum_{k=0}^{n} a_k = {}^{n+1}C_1 + {}^{n+1}C_2 + \ldots + {}^{n+1}C_{n+1} = 2^{n+1} - 1$$

DPP 7.4

Single Correct Answer Type

1. (c) ${}^nC_1 x(1-x)^{n-1} + 2 \cdot {}^nC_2 x^2(1-x)^{n-2} + 3 \cdot {}^nC_3 x^3(1-x)^{n-3} + \ldots + n \cdot {}^nC_n x^n$

$$= \sum_{r=1}^{n} r \cdot {}^nC_r x^r (1-x)^{n-r}$$

$$= \sum_{r=1}^{n} n \, {}^{n-1}C_{r-1} x^r (1-x)^{n-r}$$

$$= n \sum_{r=1}^{n} {}^{n-1}C_{r-1} x \cdot x^{r-1} (1-x)^{(n-1)-(r-1)}$$

$$= nx \sum_{r=1}^{n} {}^{n-1}C_{r-1} x^{r-1} (1-x)^{(n-1)-(r-1)}$$

$$= nx[x + (1-x)]^{n-1}$$

$$= nx$$

${}^8C_1 x(1-x)^7 + 2 \cdot {}^8C_2 x^2(1-x)^6 + 3 \cdot {}^8C_3 x^3(1-x)^5 + \ldots + 8x^8$

$= 8x$

2. (a) $99^{50} - 99.98^{50} + \frac{99.98}{1.2}(97)^{50} - \ldots + 99$

$= {}^{99}C_0 \, 99^{50} - {}^{99}C_1 (99-1)^{50} + {}^{99}C_2 (99-2)^{50} - \ldots + {}^{99}C_{98} (99-98)^{50} + {}^{99}C_{99} (99-99)^{50}$

$= 99^{50} ({}^{99}C_0 - {}^{99}C_1 + {}^{99}C_2 - {}^{99}C_3 + \ldots.) + {}^{50}C_1 \cdot 99^{49} ({}^{99}C_1 - 2 \cdot {}^{99}C_2 + 3 \cdot {}^{99}C_3 - \ldots) + \ldots$

$= 0 + 0 + 0 + \ldots + 0 = 0$

3. (c)

$k \, {}^nC_k = n \, {}^{n-1}C_{k-1}$

$\therefore \quad \sum_{k=1}^{n} k^2 ({}^nC_k)^2$

$= n^2 \sum_{k=1}^{n} ({}^{n-1}C_{k-1})^2$

$= n^2 ({}^{2n-2}C_{n-1})$

$\therefore \quad f(5) = 25({}^8C_4) = 1750$

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4. (iii) $\frac{r}{r+1}{}^nC_r = \left(1-\frac{1}{r+1}\right){}^nC_r$

$= {}^nC_r - \frac{1}{r+1}\cdot\frac{n!}{r!(n-r)!}$

$= {}^nC_r - \frac{1}{n+1}{}^{n+1}C_{r+1}$

$\therefore \sum_{r=1}^{n}(-1)^{r-1}\frac{r}{r+1}\cdot{}^nC_r$

$= {}^nC_1 - {}^nC_2 + C_3 - \ldots - \frac{1}{n+1}[{}^{n+1}C_2 - {}^{n+1}C_3 + {}^{n+1}C_4 - \ldots]$

$= {}^nC_0 - \frac{1}{n+1}[-{}^{n+1}C_0 + {}^{n+1}C_1]$

$= 1 - \frac{1}{n+1}[-1 + (n+1)]$

$= \frac{1}{n+1}$

5. (a) Writing $\binom{200}{r} = \binom{200}{200-r}$, we have

$\binom{100}{0}\binom{200}{50} + \binom{100}{1}\binom{200}{49} + \binom{100}{2}\binom{200}{48} + \ldots + \binom{100}{50}\binom{200}{0}$

= Coefficient of x^{50} in the expansion of $(1+x)^{100}(x+1)^{200}$

= Coefficient of x^{50} in the binomial expansion of $(1+x)^{300}$

$= \binom{300}{50}$

6. (b) $S_{100} = \frac{0}{({}^{100}C_0)^5} + \frac{1}{({}^{100}C_1)^5} + \frac{2}{({}^{100}C_2)^5} + \ldots + \frac{100}{({}^{100}C_{100})^5}$ (1)

Also $S_{100} = \frac{100}{({}^{100}C_0)^5} + \frac{(100-1)}{({}^{100}C_1)^5} + \frac{(100-2)}{({}^{100}C_2)^5} + \ldots + \frac{0}{({}^{100}C_{100})^5}$ (2)

$\therefore$ On adding (1) and (2), we get

$2S_{100} = 100\, t_{100}$

$\Rightarrow \quad \frac{100 t_{100}}{S_{100}} = 2$

7. (a) We have $S_1 + S_2 + S_3 = \sum_{i=0}^{100}\sum_{j=0}^{100} C_i C_j$

$= ({}^{100}C_0 + {}^{100}C_1 + {}^{100}C_2 + \ldots + {}^{100}C_{100})^2$

$= (2^{100})^2 = 2^{200}$

Now $S_1 + S_2 + S_3 = 2^{200} = 4^{100} = 16^{50} = 32^{40} = 256^{25} = \ldots = a^b$

Hence least value of $(a+b) = 16 + 50 = 66$

8. (a) ${}^{74}C_{37} - 2$

$= {}^{37}C_0{}^2 + {}^{37}C_1{}^2 + {}^{37}C_2{}^2 + \ldots {}^{37}C_{37}{}^2 - 2$

$= {}^{37}C_1{}^2 + {}^{37}C_2{}^2 + \ldots {}^{37}C_{36}{}^2$

is divisible by 37^2

(as each ${}^{37}C_1, {}^{37}C_2, {}^{37}C_3, \ldots {}^{37}C_{36}$ are divisible by 37)

9. (c) ${}^nC_0 - {}^nC_1 + {}^nC_2 - {}^nC_3 + \ldots + (-1)^r\cdot{}^nC_r$

= coefficient of x^r in the expansion of $(1-x)^n(1+x+x^2+\ldots)$

= coefficient of x^r in the expansion of $(1-x)^n(1-x)^{-1}$

= coefficient of x^r in the expansion of $(1-x)^{n-1}$

$= (-1)^r\cdot{}^{n-1}C_r$

$\Rightarrow (-1)^r\cdot{}^{n-1}C_r = 28 \Rightarrow r$ must be even

${}^{n-1}C_r = 28 \Rightarrow {}^{n-1}C_r = 7\times 4 = \frac{7\times 8}{2} = {}^8C_2 = n-1 = 8 \Rightarrow n = 9$

10. (a) $S = {}^nC_0 + 2\cdot{}^nC_1 + 3\cdot{}^nC_2 + \ldots + (n+1)\cdot{}^nC_n$

Here $T_r = (r+1){}^nC_r = n{}^{n-1}C_{r-1} + {}^nC_r$

$\therefore \quad S = n2^{n-1} + 2^n = 576$ (given)

$\therefore \quad n = 7$

11. (d) ${}^{50}C_6 - {}^5C_1{}^{40}C_6 + {}^5C_2{}^{30}C_6 - {}^5C_3{}^{20}C_6 + {}^5C_4{}^{10}C_6$

= coefficient of x^6 in $[{}^5C_0(1+x)^{50} - {}^5C_1(1+x)^{40}$

$+ {}^5C_2(1+x)^{30} - {}^5C_3(1+x)^{20} + {}^5C_4(1+x)^{10} - {}^5C_5(1+x)^0]$

= coefficient x^6 in $[(1+x)^{10} - 1]^5$

= coefficient of x^6 in $({}^{10}C_1 x + {}^{10}C_2 x^2 + \ldots)^5$

= coefficient of x in $({}^{10}C_1 + {}^{10}C_2 x + \ldots)^5$

$= {}^5C_1({}^{10}C_2)({}^{10}C_1)^4 = 2250000$

12. (a) Let the required value be S

$\sum_{j=0}^{n}\sum_{i=0}^{n}(-1)^{i+j-1}{}^nC_i\cdot{}^nC_j = \sum_{i=0}^{n}(-1)^{2i-1}({}^nC_i)^2 + 2S$

$\therefore \quad 0 = -\sum_{i=0}^{n}({}^nC_i)^2 + 2S$

$\therefore \quad 2S = {}^{2n}C_n$

$\therefore \quad S = {}^{2n-1}C_{n-1} = {}^{2n-1}C_n$

13. (b) $\sum_{m=1}^{n}{}^nC_m\left(\sum_{k=1}^{m}\left(\sum_{p=k}^{m}\frac{m!}{p!(m-p)!}\cdot\frac{p!}{k!(p-k)!}\right)\right)$

$= \sum_{m=1}^{n}{}^nC_m\left(\sum_{k=1}^{m}\left(\sum_{p=k}^{m}{}^{m-k}C_{p-k}\right)\frac{m!}{k!(m-k)!}\right)$

$= \sum_{m=1}^{n}{}^nC_m\left(\sum_{k=1}^{m}2^{m-k}\cdot{}^mC_k\right)$

$= \sum_{m=1}^{n}{}^nC_m((1+2)^m - 2^m) = \sum_{m=1}^{n}({}^nC_m 3^m - {}^nC_m 2^m)$

$= (1+3)^n - 1 - (1+2)^n + 1 = 4^n - 3^n$

Multiple Correct Answers Type

14. (a, b)

The given series can be written as

$S = \sum_{r=0}^{n}{}^nC_r\ {}^{2n-2r}C_m(-1)^r$

$= \sum_{r=0}^{n}{}^nC_r(-1)^r \times$ coefficient of x^m in $(1+x)^{2n-2r}$

= coefficient of x^m in $\sum_{r=0}^{n}{}^nC_r(-1)^r[(1+x)^2]^{n-r}$

= coefficient of x^m in $[(1+x)^2 - 1]^n$

= coefficient of x^m in $(x^2+2x)^n$

= coefficient of x^m in $x^n(x+2)^n$

= coefficient of x^{m-n} in $(x+2)^n$

$= {}^nC_{m-n}2^{n-(m-n)} = \binom{n}{m-n}2^{2n-m}$ if $m \geq n$ and 0 if $m < n$.

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15. (a, d)

$${}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - \ldots - {}^{20}C_{15}$$

= coefficient of x^{15} in the expansion of $(1-x)^{20}(1+x+x^2+\ldots)$

= coefficient of x^{15} in the expansion of $(1-x)^{20}(1-x)^{-1}$

= coefficient of x^{15} in the expansion of $(1-x)^{19}$

$= -{}^{19}C_{15}$

$$16\,{}^{20}C_0 - 15\,{}^{20}C_1 + 14\,{}^{20}C_2 - \ldots + 2\,{}^{20}C_{14} - {}^{20}C_{15}$$

= coefficient of x^{15} in the expansion of $(1-x)^{20}(1+2x+3x^2+\ldots)$

= coefficient of x^{15} in the expansion of $(1-x)^{20}(1-x)^{-2}$

= coefficient of x^{15} in the expansion of $(1-x)^{18}$

$= {}^{18}C_{15}$

16. (a, b)

$$\sum_{k=0}^{7}\left(\frac{{}^7C_k}{{}^{14}C_k}\sum_{r=k}^{14}{}^rC_k\cdot{}^{14}C_r\right)$$

$$=\sum_{k=0}^{7}\left(\frac{{}^7C_k}{14!}\times k!(14-k)!\sum_{r=k}^{14}\frac{r!}{k!(r-k)!}\cdot\frac{14!}{r!(14-r)!}\right)$$

$$=\sum_{k=0}^{7}\left({}^7C_k\sum_{r=k}^{14}{}^{14-k}C_{r-k}\right)$$

$$=\sum_{k=0}^{7}{}^7C_k\cdot 2^{14-k}=2^{14}\sum_{k=0}^{7}{}^7C_k\left(\frac{1}{2}\right)^k$$

$$=2^{14}\cdot\left(1+\frac{1}{2}\right)^7=6^7>7^6$$

17. (a, b, c, d)

$${}^{1000}C_{50} + {}^{999}C_{49} + {}^{998}C_{48} + \ldots + {}^{950}C_0$$

= coefficient of x^{950} in $\{(1+x)^{950}+(1+x)^{951}+\ldots+(1+x)^{1000}\}$

= coefficient of x^{950} in $(1+x)^{950}\{1+(1+x)+(1+x)^2+\ldots+(1+x)^{50}\}$

$$=\text{coefficient of } x^{950} \text{ in } (1+x)^{950}\,\frac{\{(1+x)^{51}-1\}}{1+x-1}$$

$$=\text{coefficient of } x^{950} \text{ in } \frac{(1+x)^{1001}-(1+x)^{950}}{x}$$

$$= {}^{1001}C_{951} = {}^{1001}C_{50} = {}^{1002}C_{951} - {}^{1001}C_{51} = {}^{1002}C_{51} - {}^{1001}C_{950},$$

Since ${}^nC_r + {}^nC_{r-1} = {}^{(n+1)}C_r$

DPP 7.5

Single Correct Answer Type

1. (b)

$$\frac{(1-3x)^{1/2}+(1-x)^{5/3}}{2\left[1-\frac{x}{4}\right]^{1/2}}$$

$$=\frac{\left[1+\frac{1}{2}(-3x)\right]+\left[1+\frac{5}{3}(-x)\right]}{2\left[1+\frac{1}{2}\left(-\frac{x}{4}\right)\right]}$$

(Neglecting higher powers of x)

$$=\frac{\left[1-\frac{19}{12}x\right]}{\left[1-\frac{x}{8}\right]}$$

$$=\left[1-\frac{19}{12}x\right]\left[1-\frac{x}{8}\right]^{-1}$$

$$=\left[1-\frac{19}{12}x\right]\left[1+\frac{x}{8}\right]=1-\frac{35}{24}x$$

then $a+bx=1-\frac{35}{24}x \Rightarrow a=1,\ b=-\frac{35}{24}$

2. (a) Let $S=1+\frac{1}{3^2}+\frac{1.4}{1.2}\frac{1}{3^4}+\frac{1.4.7}{1.2.3}\frac{1}{3^6}+\ldots$

$$\Rightarrow S=1+\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)+\frac{\frac{1}{3}\cdot\frac{4}{3}}{1.2}\left(\frac{1}{3}\right)^2+\frac{\frac{1}{3}\cdot\frac{4}{3}\cdot\frac{7}{3}}{1.2.3}\left(\frac{1}{3}\right)^3+\ldots$$

$$\Rightarrow S=1+\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)+\frac{\frac{1}{3}\left(1+\frac{1}{3}\right)}{2!}\left(\frac{1}{3}\right)^2$$

$$+\frac{\frac{1}{3}\left(1+\frac{1}{3}\right)\left(2+\frac{1}{3}\right)}{3!}\left(\frac{1}{3}\right)^3+\ldots$$

$$=\left(1-\frac{1}{3}\right)^{-\frac{1}{3}}=\left(\frac{2}{3}\right)^{-\frac{1}{3}}=\left(\frac{3}{2}\right)^{\frac{1}{3}}$$

3. (b) $P=\dfrac{1}{(1+x)(1+x^2)(1+x^4)(1+x^8)\ldots(1+x^{2^m})}$

Multiplying the numerator and denominator by $1-x$, we have given expression

$$=\frac{1-x}{(1-x)(1+x)(1+x^2)(1+x^4)\ldots(1+x^{2^m})}$$

$$=\frac{1-x}{(1-x^2)(1+x^2)(1+x^4)\ldots(1+x^{2^m})}$$

$$=\frac{1-x}{(1-x^4)(1+x^4)\ldots(1+x^{2^m})}$$

$$\ldots\ldots\ldots\ldots$$

$$=\frac{1-x}{1-x^{2^{m+1}}}=(1-x)\left(1-x^{2^{m+1}}\right)^{-1}$$

$$=(1-x)\left(1+x^{2^{m+1}}+x^{2^{m+2}}+\ldots\right)$$

4. (b) Given $(2x^2+3x+4)^{10}=\sum_{r=0}^{20}a_rx^r$

Replacing x by $\frac{2}{x}$ we get,

$$\left(\frac{8}{x^2}+\frac{6}{x}+4\right)^{10}=\sum_{r=0}^{20}a_r\left(\frac{2}{x}\right)^r$$

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$$\Rightarrow 2^{10}(2x^2+3x+4)^{10} = \sum_{r=0}^{20} a_r 2^r x^{20-r}$$

$$\Rightarrow \sum_{r=0}^{20} a_r x^r = \sum_{r=0}^{20} a_r 2^{r-10} x^{20-r}$$

Comparing coefficient x^7 both sides, we get $a_7 = a_{13} \times 2^3$.

$^{30}C_0 \cdot {}^{20}C_{10} + {}^{31}C_1 \cdot {}^{19}C_{10} + {}^{32}C_2 \cdot {}^{18}C_{10} + \ldots {}^{40}C_{10} \cdot {}^{10}C_{10}$

= coefficient of x^{10} in $(1-x)^{-11}(1-x)^{-31}$

= coefficient of x^{10} in $(1-x)^{-42}$

$= {}^{42+10-1}C_{42-1} = {}^{51}C_{41} = {}^{51}C_{10}$

Comprehension Type

6. (a) $S = \dfrac{(1+x)^n\left[1-\left(\dfrac{1+x}{2}\right)^{n+1}\right]}{1-\left(\dfrac{1+x}{2}\right)}$

$$= \frac{1}{2^n}\frac{(1+x)^n[2^{n+1}-(1+x)^{n+1}]}{1-x}$$

$$= \frac{1}{2^n}[2^{n+1}(1+x)^n - (1+x)^{2n+1}](1-x)^{-1}$$

$$= \frac{1}{2^n}[2^{n+1}(1+x)^n - (1+x)^{2n+1}](1+x+x^2+\ldots+\infty)$$

The coefficient of x^n in S

$$= \frac{1}{2^n}\left[2^{n+1}\sum_{r=0}^{n} {}^nC_r - \sum_{r=0}^{n} {}^{2n+1}C_r\right]$$

$$= \frac{1}{2^n}\left[2^{n+1}2^n - \frac{1}{2}2^{2n+1}\right]$$

$$= 2^n$$

7. (c) $\displaystyle\sum_{r=0}^{n} {}^{n+r}C_r\left(\frac{1}{2}\right)^r$

$$= {}^nC_n\left(\frac{1}{2}\right)^0 + {}^{n+1}C_n\left(\frac{1}{2}\right)^1 + {}^{n+2}C_n\left(\frac{1}{2}\right)^2 + \ldots\ldots + {}^{2n}C_n\left(\frac{1}{2}\right)^n$$

= coefficient of x^n in $(1+x)^n\left(\frac{1}{2}\right)^0 + (1+x)^{n+1}\left(\frac{1}{2}\right)^1 + (1+x)^{n+2}\left(\frac{1}{2}\right)^2 + \ldots\ldots + (1+x)^{2n}\left(\frac{1}{2}\right)^n$

= coefficient of x^n in S

$= 2^n$

8. (a), 9. (c), 10. (a)

8. (a) $S = \displaystyle\sum_{i=0}^{10} P(i, 10-i) = P(0, 10) + P(1, 9) + \ldots + P(10, 0)$

$= {}^{10}C_0 + {}^{10}C_1 + \ldots + {}^{10}C_{10} = 2^{10} = 1024$

9. (c) $P(i, 100-i) = {}^{100}C_i$

and $P(j, 100-j) = {}^{100}C_j$

Given

$\Rightarrow \quad i + j = 100$

Number of non negative integral solutions for this equation = 101 (including $i = j = 50$)

Hence required number of ordered pairs = 100

10. (a) $P(i, n-i) = {}^nC_i = P(n-i, i)$

Now $P(43, 4) + \displaystyle\sum_{j=1}^{5} P(49-j, 3)$

$= ({}^{47}C_4 + {}^{47}C_3) + {}^{48}C_3 + {}^{49}C_3 + {}^{50}C_3 + {}^{51}C_3$

$= ({}^{48}C_4 + {}^{48}C_3) + {}^{49}C_3 + {}^{50}C_3 + {}^{51}C_3$

$= ({}^{49}C_4 + {}^{49}C_3) + {}^{50}C_3 + {}^{51}C_3 = ({}^{50}C_4 + {}^{50}C_3)$

$= {}^{51}C_4 + {}^{51}C_3 = {}^{52}C_4 = P(48, 4) = P(4, 48)$

11. (b), 12. (c), 13. (b)

Let

$f(x) = (1+x)(1+x+x^2)(1+x+x^2+x^3)\ldots(1+x+x^2+\ldots+x^{20})$

Highest degree of $f(x)$

= highest degree of x present in the $f(x)$

$= 1 + 2 + 3 + \ldots + 20$

$= \dfrac{20(21)}{2} = 210$

Since in the expansion of $f(x)$ degree of x from zero to 210, all present so all terms containing $x^0, x^1, x^2 \ldots, x^{210}$ will be present

$\therefore$ total no. of terms $= 210 + 1 = 211$

$(1+x)(1+x+x^2)\ldots(1+x+x^2+\ldots+x^{20})$

$= a_0 + a_1x + a_2x^2 + \ldots + a_{210}x^{210}$ (i)

Replacing x by $1/x$, we get

$$\left(1+\frac{1}{x}\right)\left(1+\frac{1}{x}+\frac{1}{x^2}\right)\ldots\left(1+\frac{1}{x}+\frac{1}{x^2}+\ldots+\frac{1}{x^{20}}\right) = a_0 + \frac{a_1}{x} + \frac{a_2}{x^2} + \ldots + \frac{a_{210}}{x^{210}}$$

Taking L.C.M. both sides, we get

$(x+1)(x^2+x+1)\ldots(x^{20}+x^{19}+\ldots+x+1)$

$= a_0x^{210} + a_1x^{209} + \ldots + a_{209}x + a_{210}$

Thus $a_0 + a_1x + a_2x^2 + \ldots + a_{210}x^{210}$

$= a_0x^{210} + a_1x^{209} + \ldots + a_{209}x + a_{210}$

Thus $a_r = a_{210-r}$

$a_0 + a_1x + a_2x^2 + a_3x^3 + \ldots$

$= (1+x)(1+x+x^2)\ldots(1+x+x^2+\ldots+x^{20})$

Putting $x = 1$ in (i)

$\therefore \quad a_0 + a_1 + a_2 + \ldots = 21!$ (ii)

Again putting $x = -1$

$a_0 - a_1 + a_2 - a_3 + a_4 = 0$ (iii)

Adding equation (ii) and (iii), we have

$2[a_0 + a_2 + a_4 + \ldots] = 21!$

$\therefore \quad a_0 + a_2 + a_4 + \ldots = \dfrac{21!}{2}$ = sum of even coefficients

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14. (b) Differentiating the expansion we have

$$n(p+2x)(1+px+x^2)^{n-1}$$
$$= a_1 + 2a_2x + 3a_3x^2 + \ldots + 2na_{2n}x^{2n-1}$$

Multiplying by $(1+px+x^2)$

$$n(p+2x)(1+a_1x+a_2x^2+\ldots)$$
$$= (1+px+x^2)(a_1+2a_2x+3a_3x^2+\ldots+2na_{2n}x^{2n-1})$$

Comparing coefficient of x^r both side.

$$n[pa_r + 2a_{r-1}] = (r+1)a_{r+1} + pra_r + (r-1)a_{r-1}$$

$\therefore \quad (np - pr)a_r = (r+1)a_{r+1} + (r-1-2n)a_{r-1}$

15. (c) $a_1 + 5a_2 + 9a_3 + \ldots + (8n-3)a_{2n} = \sum_{r=1}^{2n}(4r-3)a_r$

$$= 4\sum_{r=1}^{2n} ra_r - 3\sum_{r=1}^{2n} a_r$$

$$(1+px+x^2)^n = 1 + a_1x + a_2x^2 + \ldots + a_{2n}x^{2n}$$

so, $\sum_{r=1}^{2n} a_r = (p+2)^n - 1$

Differentiating the expansion and substituting $x = 1$

$$\sum_{r=1}^{2n} ra_r = n(p+2)^n$$

$\therefore \quad \sum_{r=1}^{2n}(4r-3)a_r = 4n(p+2)^n - 3((p+2)^n - 1)$

$$= (4n-3)(p+2)^n + 3$$

16. (d) $a_1 + 5a_2 + 9a_3 + \ldots + (4n-1)a_{2n} = \sum_{r=1}^{2n}(2r-1)a_r$

$$s = 2\sum_{r=1}^{2n} ra_r - \sum_{r=1}^{2n} a_r$$
$$= 2n(p+2) - ((p+2)^n - 1)$$
$$= (2n-1)(p+2)^n + 1$$

now $p = -3$ and $n \in$ even

$\therefore \quad s = (2n-1) + 1 = 2n$

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CHAPTER 8

DPP 8.1

Single Correct Answer Type

1. (a) $\begin{vmatrix} 1 & \cos\alpha & \cos\beta \\ \cos\alpha & 1 & \cos\gamma \\ \cos\beta & \cos\gamma & 1 \end{vmatrix} = \begin{vmatrix} 0 & \cos\alpha & \cos\beta \\ \cos\alpha & 0 & \cos\gamma \\ \cos\beta & \cos\gamma & 0 \end{vmatrix}$

$\Rightarrow \sin^2\gamma - \cos\alpha(\cos\alpha - \cos\beta\cos\gamma) + \cos\beta(\cos\alpha\cos\gamma - \cos\beta)$

$= -\cos\alpha(-\cos\beta\cos\gamma) + \cos\beta(\cos\alpha\cos\gamma)$

$\Rightarrow \sin^2\gamma - \cos^2\alpha + 2\cos\alpha\cos\beta\cos\gamma - \cos^2\beta$

$= 2\cos\alpha\cos\beta\cos\gamma$

$\Rightarrow \sin^2\gamma = \cos^2\alpha + \cos^2\beta$

$\Rightarrow \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

2. (c) The given equation is $px^3 + qx^2 - rx - r = 0$

$$\alpha+\beta+\gamma = \frac{-q}{p},\ \alpha\beta+\beta\gamma+\gamma\alpha = -\frac{r}{p},\ \alpha\beta\gamma = \frac{r}{p}$$

$D = (1+\alpha)(1+\beta)(1+\gamma) + 1 + 1 - (1+\alpha) - (1+\beta) - (1+\gamma)$

$= \alpha\beta\gamma + \alpha\beta + \alpha\gamma + \beta\gamma$

$= \frac{r}{p} - \frac{r}{p} = 0$

3. (b) As $1 + \omega + \omega^2 = 0$

$$D = \begin{vmatrix} -\frac{x}{\omega^2} & -y & -\frac{z}{\omega} \\ -y & -\frac{z}{\omega} & -\frac{x}{\omega^2} \\ -\frac{z}{\omega} & -\frac{x}{\omega^2} & -y \end{vmatrix} = x^3 + y^3 + z^3 - 3xyz$$

$= \frac{1}{2}(x+y+z)\{(x-y)^2 + (y-z)^2 + (z-x)^2\}$

$= (x+y+z)(x+y\omega+z\omega^2)(x+y\omega^2+z\omega)$

The determinant varnishes if $x = y = z$ or $x + y\omega + z\omega^2 = 0$

4. (b) $a = \cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3} = \omega^2$

$$\therefore \begin{vmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{vmatrix} = 3(\omega - \omega^2) \text{ purely imaginary}$$

5. (b) Clearly $\alpha = -i$ where $i^2 = -1$

So $\Delta(\alpha) = 0 + \alpha^2 + 1 - 1 - 0 - \alpha^3$

$= (-i)^2 + 1 - 1 - (-i)^3$

$= -1 + 1 - 1 - i = -1 - i$

So, principal argument of $\Delta(\alpha)$ is $-\frac{3\pi}{4}$.

6. (b) $\Delta_1 = x(x^2 - ab) - b(ax - ab) + b(a^2 - ax)$

$= x^3 - 3abx + ab^2 + a^2b$

$\therefore \frac{d}{dx}(\Delta_1) = 3x^2 - 3ab = 3(x^2 - ab) = 3\Delta_2$

7. (a) We have $a^2 + b^2 + c^2 + ab + bc + ca \le 0$

$\therefore (a+b)^2 + (b+c)^2 + (c+a)^2 \le 0$

$\therefore a + b = 0,\ b + c = 0$ and $c + a = 0$

$\therefore a = b = c = 0$

$$\Rightarrow \begin{vmatrix} (a+b+2)^2 & a^2+b^2 & 1 \\ 1 & (b+c+2)^2 & b^2+c^2 \\ c^2+a^2 & 1 & (c+a+2)^2 \end{vmatrix}$$

$$= \begin{vmatrix} 4 & 0 & 1 \\ 1 & 4 & 0 \\ 0 & 1 & 4 \end{vmatrix} = 65$$

8. (a) $f(x) = \begin{vmatrix} 1+2x & 1 & 1-x \\ 2-x & 2+x & 3+x \\ x & 1+x & 1-x^2 \end{vmatrix} = 0$

Constant term $= f(0) = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 2 & 3 \\ 0 & 1 & 1 \end{vmatrix} = 2 + 2 - 2 - 3 = -1$

Also coefficient of x^4 is '–2'

Hence product of roots is 1/2.

Comprehension Type

9. (b), 10. (d)

For S_n, $a_{11}, a_{12}, a_{13}, \ldots\ldots a_{1(n-1)}$ we have two options '1' or '–1', but for a_{1n} we have only one way depending upon the product $(a_{11} \cdot a_{12} \cdot a_{13} \cdot \ldots \cdot a_{1(n-1)})$

$\therefore$ For R_1 we have 2^{n-1} ways

Similarly for $R_2, R_3, R_4, \ldots\ldots R_{n-1}$ we have 2^{n-1} ways

For R_n we have only one way.

Hence total number of ways $(2^{n-1})^{n-1} = 2^{(n-1)^2}$

For S_3, we have $2^{(3-1)^2} = 16$ elements.

DPP 8.2

Single Correct Answer Type

1. (c) $\Delta = \begin{vmatrix} 1+x & 1 & 1 \\ 1+y & 1+2y & 1 \\ 1+z & 1+z & 1+3z \end{vmatrix} = 0$

Applying $C_1 \to C_1 - C_3$, $C_2 \to C_2 - C_3$ we get

$$\Delta = \begin{vmatrix} x & 0 & 1 \\ y & 2y & 0 \\ -2z & -2z & 1+3z \end{vmatrix} = 0$$

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$$\therefore \quad \Delta=\begin{vmatrix} x & 0 & 0 \\ y & 2y & 1-\frac{y}{x} \\ -2z & -2z & 1+3z+\frac{2z}{x} \end{vmatrix}=0\left(\text{by } C_3\to C_3-\frac{1}{x}C_1\right)$$

$$\therefore \quad x\left[2y\left(1+3z+\frac{2z}{x}\right)+2z\left(1-\frac{y}{x}\right)\right]=0$$

$$\therefore \quad [2xy+6zxy+4yz+2zx-2yz]=0$$

$$\therefore \quad 2(xyz)\left[\frac{1}{x}+\frac{1}{y}+\frac{1}{z}+3\right]=0$$

$$\therefore \quad -\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)=3$$

2. (c) $\Delta=\begin{vmatrix} X & SX & tX \\ X_1 & SX_1+S_1X & tX_1+t_1X \\ X_2 & SX_2+2S_1X_1+S_2X & tX_2+2t_1X_1+t_2X \end{vmatrix}$

$\begin{pmatrix} C_2\to C_2-SC_1 \\ C_3\to C_3-tC_1 \end{pmatrix}$

$$=\begin{vmatrix} X & 0 & 0 \\ X_1 & S_1X & t_1X \\ X_2 & 2S_1X_1+S_2X & 2t_1X_1+t_2X \end{vmatrix}$$

$$=X^2\begin{vmatrix} S_1 & t_1 \\ 2S_1X_1+S_2X & 2t_1X_1+t_2X \end{vmatrix}$$

$$=X^3\begin{vmatrix} S_1 & t_1 \\ S_2 & t_2 \end{vmatrix}(R_2\to R_2-2X_1R_1)$$

$\therefore \quad n=3$

3. (a) Applying $C_1\to C_1+C_2+C_3$,

We get $\Delta=\begin{vmatrix} x+w^2+w+1 & w & 1 \\ w+w^2+1+x & w^2 & 1+x \\ 1+x+w+w^2 & x+w & w^2 \end{vmatrix}$

$$=\begin{vmatrix} x & w & 1 \\ x & w^2 & 1+x \\ x & x+w & w^2 \end{vmatrix}$$

[using $1+w+w^2=0$]

Δ is clearly equal to 0 for $x=0$.

4. (c) $|A|=\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

$$|B|=\begin{vmatrix} a_{11} & k^{-1}a_{12} & k^{-2}a_{13} \\ ka_{21} & a_{22} & k^{-1}a_{23} \\ k^2a_{31} & ka_{32} & a_{33} \end{vmatrix}$$

$$=\frac{1}{k^3}\begin{vmatrix} k^2a_{11} & ka_{12} & a_{13} \\ k^2a_{21} & ka_{22} & a_{33} \\ k^2a_{31} & ka_{32} & a_{33} \end{vmatrix}$$

$=|A|$

$k_1|A|+k_2|B|=0$

$\therefore \quad k_1+k_2=0$

5. (d) We have $\beta\gamma+\gamma\alpha+\alpha\beta=0$

$$\Delta=\frac{1}{\alpha^3\beta^3\gamma^3}\begin{vmatrix} \beta\gamma & \gamma\alpha & \alpha\beta \\ \gamma\alpha & \alpha\beta & \beta\gamma \\ \alpha\beta & \beta\gamma & \gamma\alpha \end{vmatrix}$$

$$=\frac{1}{\alpha^3\beta^3\gamma^3}\begin{vmatrix} \beta\gamma+\gamma\alpha+\alpha\beta & \gamma\alpha & \alpha\beta \\ \gamma\alpha+\alpha\beta+\beta\gamma & \alpha\beta & \beta\gamma \\ \alpha\beta+\beta\gamma+\gamma\alpha & \beta\gamma & \gamma\alpha \end{vmatrix}$$

[using $C_1\to C_1+C_2+C_3$]

$$=\frac{1}{\alpha^3\beta^3\gamma^3}\begin{vmatrix} 0 & \gamma\alpha & \alpha\beta \\ 0 & \alpha\beta & \beta\gamma \\ 0 & \beta\gamma & \gamma\alpha \end{vmatrix}=0$$ [all zero property].

6. (c) $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$

Applying $R_1\to R_1-2R_2+R_3$

$$\begin{vmatrix} 0 & 0 & 1 \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix} \qquad (\because a-2b+c=1)$$

Applying $R_3\to R_3-R_2$,
the determinant reduces to

$$\begin{vmatrix} 0 & 0 & 1 \\ x+2 & x+3 & x+b \\ 1 & 1 & c-b \end{vmatrix}=-1$$

7. (a) $\Delta=x^ky^kz^k\begin{vmatrix} 1 & ar & a^2r^2 \\ 1 & ar^2 & a^2r^4 \\ 1 & ar^3 & a^2r^6 \end{vmatrix}$

$$=a^{3k}\cdot r^{6k}\cdot a^3r^3\begin{vmatrix} 1 & 1 & 1 \\ 1 & r & r^2 \\ 1 & r^2 & r^4 \end{vmatrix}$$

$$=a^{3(k+1)}\cdot r^{6k+3}\cdot(1-r)(r-r^2)(r^2-1)$$

Clearly, $k=-1$

$\therefore \quad \Delta=r^{-2}(1-r)^2(r^2-1)$

$$=(r-1)^2\left(1-\frac{1}{r^2}\right)$$

8. (c) We have

$(a^2+b^2+c^2)x^2-2(ab+bc+cd)x+b^2+c^2+d^2\le 0$

$\Rightarrow \quad (ax-b)^2+(bx-c)^2+(cx-d)^2\le 0$

$\Rightarrow \quad (ax-b)^2+(bx-c)^2+(cx-d)^2=0$

$\Rightarrow \quad \frac{b}{a}=\frac{c}{b}=\frac{d}{c}=x$

$\Rightarrow \quad b^2=ac$ or $2\log b=\log a+\log c$,

Now, $\begin{vmatrix} 33 & 14 & \log a \\ 65 & 27 & \log b \\ 97 & 40 & \log c \end{vmatrix}$ [Apply $R_1\to R_1+R_3-2R_2$]

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$$= \begin{vmatrix} 0 & 0 & 0 \\ 65 & 27 & \log b \\ 97 & 40 & \log c \end{vmatrix}$$

9. (a) $\begin{vmatrix} {}^{x}C_r & {}^{x}C_{r+1} & {}^{x}C_{r+2} \\ {}^{y}C_r & {}^{y}C_{r+1} & {}^{y}C_{r+2} \\ {}^{z}C_r & {}^{z}C_{r+1} & {}^{z}C_{r+2} \end{vmatrix} - \begin{vmatrix} {}^{x}C_r & {}^{x+1}C_{r+1} & {}^{x+2}C_{r+2} \\ {}^{y}C_r & {}^{y+1}C_{r+1} & {}^{y+2}C_{r+2} \\ {}^{z}C_r & {}^{z+1}C_{r+1} & {}^{z+2}C_{r+2} \end{vmatrix}$

In first determinant apply $C_3 \to C_3 + C_2$ and $C_2 \to C_2 + C_1$ and then again $C_3 \to C_3 + C_2$

10. (c) Given $\begin{vmatrix} {}^{9}C_4 & {}^{9}C_5 & {}^{10}C_r \\ {}^{10}C_6 & {}^{10}C_7 & {}^{11}C_{r+2} \\ {}^{11}C_8 & {}^{11}C_9 & {}^{12}C_{r+4} \end{vmatrix} = 0$

Apply $C_2 \to C_1 + C_2$

$$\begin{vmatrix} {}^{9}C_4 & {}^{10}C_5 & {}^{10}C_r \\ {}^{10}C_6 & {}^{11}C_7 & {}^{11}C_{r+2} \\ {}^{11}C_8 & {}^{12}C_9 & {}^{12}C_{r+4} \end{vmatrix} = 0$$

Value of the determinant = 0

$\Rightarrow$ Column 2 is same as column 3

$\Rightarrow \quad r = 5$

11. (a) Applying $C_2 \to C_2 - C_1$ and then $C_3 \to C_3 - C_2$

$$\Delta = \begin{vmatrix} 1 & \sin P & \sin^2 P \\ 1 & \sin Q & \sin^2 Q \\ 1 & \sin R & \sin^2 Q \end{vmatrix} = (\sin P - \sin Q)(\sin Q - \sin P)$$

$(\sin R - \sin P)$

$\Rightarrow \quad \Delta = 0$

12. (c) $\begin{vmatrix} b\cos C & a & c\cos B \\ c\cos A & b & a\cos C \\ a\cos B & c & b\cos A \end{vmatrix}$

Apply $C_1 \to C_1 + C_3$ and using projection rule,

$$D = \begin{vmatrix} b\cos C & a & c\cos B + b\cos C \\ c\cos A & b & a\cos C + c\cos A \\ a\cos B & c & b\cos A + a\cos B \end{vmatrix}$$

$$= \begin{vmatrix} b\cos C & a & a \\ c\cos A & b & b \\ a\cos B & c & c \end{vmatrix} = 0$$

13. (c) $a = 2^n - 1, b = \dfrac{3^n - 1}{2}, c = \dfrac{5^n - 1}{4}$

$$\therefore \quad \Delta = \begin{vmatrix} a & 2b & 4c \\ 2 & 2 & 2 \\ 2^n & 3^n & 5^n \end{vmatrix}$$

$$\Delta = 2 \cdot \begin{vmatrix} 2^n - 1 & 3^n - 1 & 5^n - 1 \\ 1 & 1 & 1 \\ 2^n & 3^n & 5^n \end{vmatrix} = 0$$

14. (b) We have $D = \begin{vmatrix} a_1 & a_2 & a_3 \\ 5 & 4 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix}$

Since $a_n = \dfrac{20}{n}$; $d = \dfrac{1}{20}$

Hence $D = \begin{vmatrix} 20 & \frac{20}{2} & \frac{20}{3} \\ \frac{20}{4} & \frac{20}{5} & \frac{20}{6} \\ \frac{20}{7} & \frac{20}{8} & \frac{20}{9} \end{vmatrix} = \frac{(20)^3}{4\times 7}\begin{vmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ 1 & \frac{4}{5} & \frac{2}{3} \\ 1 & \frac{7}{8} & \frac{7}{9} \end{vmatrix}$

Applying $R_1 \to R_1 - R_2$ and $R_2 \to R_2 - R_3$

$$= \frac{(20)^3}{4\times 7}\begin{vmatrix} 0 & \frac{-3}{10} & \frac{-1}{3} \\ 0 & \frac{-3}{40} & \frac{-1}{9} \\ 1 & \frac{7}{8} & \frac{7}{9} \end{vmatrix}$$

$$= \frac{50}{21}$$

15. (a) Multiplying C_1 by a, C_2 by b and C_3 by c, we obtain

$$\Delta = \frac{1}{abc}\begin{vmatrix} \frac{a}{c} & \frac{b}{c} & -\frac{a+b}{c} \\ -\frac{b+c}{a} & \frac{b}{a} & \frac{c}{a} \\ -\frac{b(b+c)}{ac} & \frac{b(a+2b+c)}{ac} & -\frac{b(a+b)}{ac} \end{vmatrix}$$

Applying $C_1 \to C_1 + C_2 + C_3$ we get

$$\Delta = \frac{1}{abc}\begin{vmatrix} 0 & \frac{b}{c} & -\frac{a+b}{c} \\ 0 & \frac{b}{a} & \frac{c}{a} \\ 0 & \frac{b(a+2b+c)}{ac} & -\frac{b(a+b)}{ac} \end{vmatrix}$$

This shows that Δ is independent of a, b and c.

DPP 8.3

Single Correct Answer Type

1. (d) $\begin{vmatrix} (1+x)^2 & (1-x)^2 & -(2+x^2) \\ 2x+1 & 3x & 1-5x \\ x+1 & 2x & 2-3x \end{vmatrix} + \begin{vmatrix} (1+x)^2 & 2x+1 & x+1 \\ (1-x)^2 & 3x & 2x \\ 1-2x & 3x-2 & 2x-3 \end{vmatrix} = 0$

$$\Rightarrow \begin{vmatrix} (1+x)^2 & 2x+1 & x+1 \\ (1-x)^2 & 3x & 2x \\ -(2+x^2) & 1-5x & 2-3x \end{vmatrix}$$

$$+ \begin{vmatrix} (1+x)^2 & 2x+1 & x+1 \\ (1-x)^2 & 3x & 2x \\ 1-2x & 3x-2 & 2x-3 \end{vmatrix} = 0$$

(taking transpose of 1st determinant)

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$$\Rightarrow \begin{vmatrix} (1+x)^2 & 2x+1 & x+1 \\ (1-x)^2 & 3x & 2x \\ -(2+x^2)+1-2x & 1-5x+3x-2 & 2-3x+2x-3 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} (1+x)^2 & 2x+1 & x+1 \\ (1-x)^2 & 3x & 2x \\ -(1+x)^2 & -2x-1 & -1-x \end{vmatrix} = 0$$

Here 1st row and 3rd row are the same
$\Rightarrow$ This is an identity
$\Rightarrow$ Infinite roots

2. (d) $\Delta_1 = \begin{vmatrix} ap^2 & 2ap & 1 \\ aq^2 & 2aq & 1 \\ ar^2 & 2ar & 1 \end{vmatrix}$

$$= 2a^2 \begin{vmatrix} p^2 & p & 1 \\ q^2 & q & 1 \\ r^2 & r & 1 \end{vmatrix}$$

$$= -2a^2(p-q)(q-r)(r-p)$$

$$\Delta_2 = \begin{vmatrix} apq & a(p+q) & 1 \\ aqr & a(q+r) & 1 \\ arp & a(r+p) & 1 \end{vmatrix}$$

$$= a^2 \begin{vmatrix} pq & (p+q) & 1 \\ qr & (q+r) & 1 \\ rp & (r+p) & 1 \end{vmatrix}$$

$$= a^2 \begin{vmatrix} pq-qr & p-r & 0 \\ qr-rp & q-p & 0 \\ rp & (r+p) & 1 \end{vmatrix} \quad (R_1 \to R_1 - R_2, R_2 \to R_2 - R_3)$$

$$= a^2(p-q)(r-p) \begin{vmatrix} -q & -1 & 0 \\ -r & -1 & 0 \\ rp & (r+p) & 1 \end{vmatrix}$$

$$= a^2(p-q)(r-p) \begin{vmatrix} -q+r & 0 & 0 \\ -r & -1 & 0 \\ rp & (r+p) & 1 \end{vmatrix}$$

$$= a^2(p-q)(r-p)(q-r)$$

$\Rightarrow \Delta_1 + 2\Delta_2 = 0$

3. (d) $\begin{vmatrix} (1+ap)^2 & (1+bp)^2 & (1+cp)^2 \\ (1+aq)^2 & (1+bq)^2 & (1+cq)^2 \\ (1+ar)^2 & (1+br)^2 & (1+cr)^2 \end{vmatrix}$

$$= \begin{vmatrix} 1 & 2a & a^2 \\ 1 & 2b & b^2 \\ 1 & 2c & c^2 \end{vmatrix} \times \begin{vmatrix} 1 & p & p^2 \\ 1 & q & q^2 \\ 1 & r & r^2 \end{vmatrix}$$

$$= 2 \times 2\Delta_1 \cdot 2\Delta_2$$

$$= 8\Delta_1\Delta_2 = 8 \times \frac{1}{2} \times 4 = 16$$

4. (a) $\frac{1}{\alpha\beta\gamma} \begin{vmatrix} \alpha\beta\gamma & \alpha\beta\gamma' + \alpha\beta'\gamma & \alpha\beta'\gamma' \\ \alpha\beta\alpha & \beta\gamma\alpha' + \beta\gamma'\alpha & \beta\gamma'\alpha' \\ \alpha\beta\gamma & \alpha\beta'\gamma + \alpha'\beta\gamma & \alpha'\beta'\gamma' \end{vmatrix}$

Multiplying R_1 by α, R_2 by β and R_3 by γ and dividing the determinant by $\alpha\beta\gamma$ we have

$$= \frac{1}{\alpha\beta\gamma} \cdot \alpha\beta\gamma \begin{vmatrix} 1 & \alpha\beta\gamma' + \alpha\beta'\gamma & \alpha\beta'\gamma' \\ 1 & \beta\gamma\alpha' + \beta\alpha\gamma' & \beta\gamma'\alpha' \\ 1 & \gamma\alpha\beta' + \gamma\alpha'\beta & \gamma\alpha'\beta' \end{vmatrix}$$

by $R_3 \to R_3 - R_1$
$R_2 \to R_2 - R_1$

$$= \begin{vmatrix} 1 & \alpha\beta\gamma' + \alpha\beta'\gamma & \alpha\beta'\gamma' \\ 0 & \gamma(\alpha'\beta - \alpha\beta') & \gamma'(\alpha'\beta - \alpha\beta') \\ 0 & \beta(\alpha'\gamma - \alpha\gamma') & \beta'(\alpha'\gamma - \alpha\gamma') \end{vmatrix}$$

$$= (\alpha'\beta - \alpha\beta')(\alpha'\gamma - \alpha\gamma') \begin{vmatrix} 1 & \alpha\beta\gamma' + \alpha\beta'\gamma & \alpha\beta'\gamma' \\ 0 & \gamma & \gamma' \\ 0 & \beta & \beta' \end{vmatrix}$$

$$= (\alpha'\beta - \alpha\beta')(\alpha'\gamma - \alpha\gamma')(\gamma\beta' - \beta\gamma')$$

5. (d) $P = \begin{vmatrix} c-a & c-b & ab-c^2 \\ a-b & a-c & bc-a^2 \\ b-c & b-a & ca-b^2 \end{vmatrix} = \begin{vmatrix} a & b & 1 \\ b & c & 1 \\ c & a & 1 \end{vmatrix}^2 = (2010)^2$

$$= (2 \times 3 \times 5 \times 67)^2 = 2^2 3^2 5^2 (67)^2$$

No. of divisors of $P = (2+1)(2+1)(2+1)(2+1) = 81$

6. (c) $\Delta\Delta^2 = 64$

$\Rightarrow \Delta^3 = 64 \Rightarrow \Delta = 4$

$$\begin{vmatrix} 2a+3l & 3l+5m & 5m+4a \\ 2l+3b & 3b+5n & 5n+4l \\ 2m+3n & 3n+5c & 5c+4m \end{vmatrix}$$

$$= [(2 \times 3 \times 5) + (3 \times 5 \times 4)]\Delta$$
$$= (30 + 60)\Delta$$
$$= 90(4)$$
$$= 360$$

7. (d) $\Delta = \begin{vmatrix} x & y & 0 \\ a & b & 0 \\ 1 & 1 & 0 \end{vmatrix} \cdot \begin{vmatrix} x & y & 0 \\ a & b & 0 \\ 1 & 1 & 0 \end{vmatrix} = 0$

8. (b) $\Delta_1\Delta_2 = \begin{vmatrix} ax+by+cz & ay+bz+cx & az+bx+cy \\ bx+cy+az & by+cz+ax & bz+cx+ay \\ cx+ay+bz & cy+az+bx & cz+ax+by \end{vmatrix}$

$$= \begin{vmatrix} u & v & w \\ w & u & v \\ v & w & u \end{vmatrix}$$

$$= u(u^2 - vw) - v(wu - v^2) + w(w^2 - uv)$$
$$= u^3 + v^3 + w^3 - 3uvw$$

9. (a) As $u + v + w = n$ and $u, v, w \geq 1$
Now, number of solutions of $u + v + w = n \Rightarrow P_n = {}^{n-1}C_{n-3}$
Similarly $P_{n+1} = {}^{n}C_{n-2}$; $P_{n+2} = {}^{n+1}C_{n-1}$; $P_{n+3} = {}^{n+2}C_n$; $P_{n+4} = {}^{n+3}C_{n+1}$.

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$$\text{Now } \Delta = \begin{vmatrix} {}^{n-1}C_{n-3} & {}^{n}C_{n-2} & {}^{n+1}C_{n-1} \\ {}^{n}C_{n-2} & {}^{n+1}C_{n-1} & {}^{n+2}C_{n} \\ {}^{n+1}C_{n-1} & {}^{n+2}C_{n} & {}^{n+3}C_{n+1} \end{vmatrix}$$

$$= \frac{1}{8}\begin{vmatrix} \frac{(n-1)!}{(n-3)!} & \frac{n!}{(n-2)!} & \frac{(n+1)!}{(n-1)!} \\ \frac{n!}{(n-2)!} & \frac{(n+1)!}{(n-1)!} & \frac{(n+2)!}{n!} \\ \frac{(n+1)!}{(n-1)!} & \frac{(n+2)!}{n!} & \frac{(n+3)!}{(n+1)!} \end{vmatrix}$$

$$= \frac{1}{8}\begin{vmatrix} (n-1)(n-2) & n(n-1) & n(n+1) \\ n(n-1) & n(n+1) & (n+1)(n+2) \\ n(n+1) & (n+2)(n+1) & (n+3)(n+2) \end{vmatrix}$$

$$\Delta = \frac{1}{2}\begin{vmatrix} 1 & n(n-1) & n \\ 1 & n(n+1) & (n+1) \\ 1 & (n+2)(n+1) & (n+2) \end{vmatrix}$$

(On applying (first) $C_3 \to C_3 - C_2$ and $C_1 \to C_1 - C_2$ (and then) $C_1 \to C_1 + C_3$)

$$= \frac{1}{2}\begin{vmatrix} 1 & n(n-1) & n \\ 0 & 2n & 1 \\ 0 & 2(n+1) & 1 \end{vmatrix} (R_3 \to R_3 - R_2 \text{ and } R_2 \to R_2 - R_1) \Rightarrow \Delta = -1$$

10. (a) Differentiating given equation w.r.t. x, we get

$$\begin{vmatrix} f'(x) & g'(x) & h'(x) \\ a & b & c \\ p & q & r \end{vmatrix} = 4mx^3 + 3nx^2 + 2rx + 5 \quad (1)$$

Again differentiating w.r.t. x, we get

$$\begin{vmatrix} f''(x) & g''(x) & h''(x) \\ a & b & c \\ p & q & r \end{vmatrix} = 12mx^2 + 6nx + 2r \quad (2)$$

Again differentiating w.r.t. x, we get

$$\begin{vmatrix} f'''(x) & g'''(x) & h'''(x) \\ a & b & c \\ p & q & r \end{vmatrix} = 24mx + 6n \quad \ldots(3)$$

Putting $x = 0$ in (2), we get

$$2r = \begin{vmatrix} f''(0) & g''(0) & h''(0) \\ a & b & c \\ p & q & r \end{vmatrix} \quad (4)$$

Putting $x = 0$ in (3), we get

$$6n = \begin{vmatrix} f'''(0) & g'''(0) & h'''(0) \\ a & b & c \\ p & q & r \end{vmatrix} \quad (5)$$

From (5) and (4), we get

$$\begin{vmatrix} f'''(0) - f''(0) & g'''(0) - g''(0) & h'''(0) - h''(0) \\ a & b & c \\ p & q & r \end{vmatrix} = 2(3n - r)$$

11. (b) Note that $\Delta(x)$ is a polynomial of degree at most 6 in x.
If $\Delta(x) = a_0 + a_1x + a_2x^2 + \ldots + a_6x^6$, then $\Delta'(x) = a_1 + 2a_2x + \ldots + 6a_6x^5$
$\Rightarrow \quad a_1 = \Delta'(0)$,
Now,

$$\Delta'(x) = \begin{vmatrix} 1 & (x-1)^2 & x^3 \\ 1 & x^2 & (x+1)^3 \\ 1 & (x+1)^2 & (x+2)^3 \end{vmatrix}$$

$$+ \begin{vmatrix} x-2 & 2(x-1) & x^3 \\ x-1 & 2x & (x+1)^3 \\ x & 2(x+1) & (x+2)^3 \end{vmatrix}$$

$$+ \begin{vmatrix} x-2 & (x-1)^2 & 3x^2 \\ x-1 & x^2 & 3(x+1)^2 \\ x & (x+1)^2 & 3(x+2)^2 \end{vmatrix}$$

$$\Rightarrow \quad \Delta'(0) = \begin{vmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 8 \end{vmatrix} + \begin{vmatrix} -2 & -2 & 0 \\ -1 & 0 & 1 \\ 0 & 2 & 8 \end{vmatrix} + \begin{vmatrix} -2 & 1 & 0 \\ -1 & 0 & 3 \\ 0 & 1 & 12 \end{vmatrix}$$

$$= -8 - 12 + 18 = -2$$

12. (d) $f(x) = \begin{vmatrix} \sin x & \cos x & \tan x \\ x^3 & x^2 & x \\ 2x & 1 & x \end{vmatrix}$

$$\therefore \quad \frac{f(x)}{x^2} = \begin{vmatrix} \frac{\sin x}{x} & \cos x & \frac{\tan x}{x} \\ x^2 & x^2 & 1 \\ 2 & 1 & 1 \end{vmatrix}$$

$$\therefore \quad \lim_{x \to 0} \frac{f(x)}{x^2} = \begin{vmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 2 & 1 & 1 \end{vmatrix} = 1$$

Multiple Correct Answers Type

13. (a, b, c)

$$\begin{vmatrix} a_1 + b_1x & a_1x + b_1 & c_1 \\ a_2 + b_2x & a_2x + b_2 & c_2 \\ a_3 + b_3x & a_3x + b_3 & c_3 \end{vmatrix} = 0$$

$$\Rightarrow \quad \underbrace{\begin{vmatrix} a_1 & a_1x + b_1 & c_1 \\ a_2 & a_2x + b_2 & c_2 \\ a_3 & a_3x + b_3 & c_3 \end{vmatrix}}_{D_1} + \underbrace{\begin{vmatrix} b_1x & a_1x + b_1 & c_1 \\ b_2x & a_2x + b_2 & c_2 \\ b_3x & a_3x + b_3 & c_3 \end{vmatrix}}_{D_2} = 0$$

Applying $C_2 \to C_2 - xC_1$ in D_1 and taking x common from C_1 in D_2

$$\Rightarrow \quad \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + x\begin{vmatrix} b_1 & a_1x + b_1 & c_1 \\ b_2 & a_2x + b_2 & c_2 \\ b_3 & a_3x + b_3 & c_3 \end{vmatrix} = 0$$

Applying $C_2 \to C_2 - C_1$ and then taking x common from C_2 in D_2

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$$\Rightarrow \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + x^2 \begin{vmatrix} b_1 & a_1 & c_1 \\ b_2 & a_2 & c_2 \\ b_3 & a_3 & c_3 \end{vmatrix} = 0$$

$$\Rightarrow (1-x^2) \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

$$\Rightarrow x = \pm 1, \text{ or } \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

14. (a, c, d)

Let $a_1 = (2m+1)^2$, $a_2 = (2n+1)^2$

$\Rightarrow \quad a_1 - a_2 = 4(m(m+1) - n(n+1)) = 8k$

so, difference of any two odd square is always a multiple of 8

Now apply $C_1 - C_3$ and $C_2 - C_3$, then C_1 and C_2 both become multiple of 8 so Δ always a multiple of 64.

15. (a, c)

$$\begin{vmatrix} \cos(\theta+\alpha) & -\sin(\theta+\alpha) & \cos 2\alpha \\ \sin\theta & \cos\theta & \sin\alpha \\ -\cos\theta & \sin\theta & \lambda\cos\alpha \end{vmatrix}$$

$$= \frac{1}{\sin\alpha\cos\alpha} \begin{vmatrix} \cos(\theta+\alpha) & -\sin(\theta+\alpha) & \cos 2\alpha \\ \sin\theta\sin\alpha & \cos\theta\sin\alpha & \sin^2\alpha \\ -\cos\theta\cos\alpha & \sin\theta\cos\alpha & \lambda\cos^2\alpha \end{vmatrix}$$

[Multiplying R_2 and R_3 by $\sin\alpha$ and $\cos\alpha$, respectively]

$$= \frac{1}{\sin\alpha\cos\alpha} \times \begin{vmatrix} 0 & 0 & \cos 2\alpha + \sin^2\alpha + \lambda\cos^2\alpha \\ \sin\theta\sin\alpha & \cos\theta\sin\alpha & \sin^2\alpha \\ -\cos\theta\cos\alpha & \sin\theta\cos\alpha & \lambda\cos^2\alpha \end{vmatrix}$$

[Applying $R_1 \to R_1 + R_2 + R_3$]

$$= \frac{\cos 2\alpha + \sin^2\alpha + \lambda\cos^2\alpha}{\sin\alpha\cdot\cos\alpha} \begin{vmatrix} \sin\theta\sin\alpha & \cos\theta\sin\alpha \\ -\cos\theta\cos\alpha & \sin\theta\cos\alpha \end{vmatrix}$$

$$= (\cos^2\alpha + \lambda\cos^2\alpha) \begin{vmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{vmatrix} = (1+\lambda)\cos^2\alpha$$

Therefore, the given determinant is independent of θ for all real values of λ.

Also, $\lambda = -1$, then it is independent of θ and α.

DPP 8.4

Single Correct Answer Type

1. (c) The system of linear equation has a non zero solution.

$$\Delta = \begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} = 0$$

$\therefore \quad (3bc - 4bc) - (2ac - 4ac) + (2ab - 3ab) = 0$

$\therefore \quad bc + ab = 2ac$

$\therefore \quad \frac{1}{a} + \frac{1}{c} = \frac{2}{b}$

$\Rightarrow a, b, c$ are in H.P.

2. (d) $\begin{vmatrix} \lambda-1 & 3\lambda+1 & 2\lambda \\ \lambda-1 & 4\lambda-2 & \lambda+3 \\ 2 & 3\lambda+1 & 3(\lambda-1) \end{vmatrix} = 0$

$\Rightarrow \lambda = 0$ or 3

If $\lambda = 0$, the equations become

$-x + y = 0$,

$-x - 2y + 3z = 0$ and

$2x + y - 3z = 0$,

$\therefore \quad \frac{x}{6-3} = \frac{y}{6-3} = \frac{z}{-1+4}$

3. (c) To have a non-trivial solution

$$\begin{vmatrix} t & t+1 & t-1 \\ t+1 & t & t+2 \\ t-1 & t+2 & t \end{vmatrix} = 0$$

$$\Rightarrow \quad 2t+1 = 0 \Rightarrow t = -\frac{1}{2}$$

4. (d) The given system of equations will have a non-trivial solution if

$$\begin{vmatrix} \alpha+a & \alpha & \alpha \\ \alpha & \alpha+b & \alpha \\ \alpha & \alpha & \alpha+c \end{vmatrix} = 0$$

Operate $R_2 \to R_2 - R_1$; $R_3 - R_3 - R_1$, then

$$\begin{vmatrix} \alpha+a & \alpha & \alpha \\ -a & b & 0 \\ -a & 0 & c \end{vmatrix} = 0$$

$\Rightarrow \quad \alpha(bc + ca + ab) + abc = 0$

Since, $a, b, c \neq 0$

$$\therefore \quad \frac{1}{\alpha} = -\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$$

5. (d) For non trivial solution $\begin{vmatrix} \sin\theta & -\cos\theta & \lambda+1 \\ \cos\theta & \sin\theta & -\lambda \\ \lambda & \lambda+1 & \cos\theta \end{vmatrix} = 0$

$\Rightarrow \sin^2\theta\cos\theta + \lambda^2\cos\theta + (\lambda+1)^2\cos\theta - \sin\theta\lambda(\lambda+1) + \cos^3\theta + \sin\theta\lambda(\lambda+1) = 0$

$\Rightarrow (\sin^2\theta + \cos^2\theta)\cos\theta + \lambda^2\cos\theta + (\lambda+1)^2\cos\theta = 0$

$\Rightarrow \cos\theta + \lambda^2\cos\theta + (\lambda+1)^2\cos\theta = 0$

$\Rightarrow 2\cos\theta(\lambda^2 + \lambda + 1) = 0$

$\Rightarrow \cos\theta = 0$

$\Rightarrow \theta = (2n+1)\frac{\pi}{2}, \lambda \in R, n \in I$

6. (b) $\Delta = \begin{vmatrix} 1 & -2 & 1 \\ 2 & 1 & -2 \\ 1 & 3 & -3 \end{vmatrix} = 0$

Hence for atleast one solution if $\Delta_1 = \Delta_2 = \Delta_3 = 0$

$$\therefore \quad \Delta_1 = \begin{vmatrix} a & -2 & 1 \\ b & 1 & -2 \\ c & 3 & -3 \end{vmatrix} = 0$$

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$\Rightarrow a - b + c = 0$

From $\Delta_2 = 0$ and $\Delta_3 = 0$, we get the same condition.

7. (b) Let $\Delta = \begin{vmatrix} \sin A & 1 & 1 \\ 1 & \sin B & 1 \\ 1 & 1 & \sin C \end{vmatrix}$

Apply $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$

Then expand along C_1, we get

$\Delta = \sin A(1 - \sin B)(1 - \sin C) + (1 - \sin A)(1 - \sin B) + (1 - \sin A)(1 - \sin C)$

Since A, B, C are angles of a triangle, $0 < \sin A, \sin B, \sin C \le 1$

$\Rightarrow \Delta \neq 0$

Multiple Correct Answers Type

8. (a, b)

The given system of equations is

$x + 2y + z = 1$ (1)

$x + 3y + 4z = k$ (2)

$x + 5y + 10z = k^2$ (3)

Subtracting (1) from (2), we get $y + 3z = k - 1$

Subtracting (2) from (3), we get $2y + 6z = k^2 - k$

$\Rightarrow 2(k - 1) = k^2 - k$

$\Rightarrow k^2 - 3k + 2 = 0$

$\Rightarrow k = 1, k = 2$

For $k = 1$;

$y + 3z = 0$ and $y = -3z$

$\Rightarrow x - 6z + z = 1$ (from (1))

$\Rightarrow x = 1 + 5z$

One set of solution $(1 + 5\lambda, -3\lambda, \lambda)$ where λ is a variable parameter.

For $k = 2$

$y + 3z = 1$ and $y = 1 - 3z$

$x + 2 - 6z + z = 1$ (from (1))

$\Rightarrow x = 5z - 1$

$\therefore$ Another set of solution $(5\lambda - 1, 1 - 3\lambda, \lambda)$ where λ is a variable parameter.

9. (b, c, d)

For the system to have nontrivial solution

$$\begin{vmatrix} \sin\theta & -2\cos\theta & -a \\ -1 & 1 & 1 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

i.e. $\sin\theta + 4\cos\theta = 3a$

$\Rightarrow -\frac{\sqrt{17}}{3} \le a \le \frac{\sqrt{17}}{3}$

$\Rightarrow a$ has three integer values.

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CHAPTER 9

DPP 9.1

Single Correct Answer Type

1. (d) $AB = BA = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$

2. (b) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 9 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 9 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 7 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 16 \\ 0 & 1 \end{bmatrix}$

If n is no. of matrices that are multiplied, then product is $\begin{bmatrix} 1 & n^2 \\ 0 & 1 \end{bmatrix}$

$\therefore \quad r = 2500$

3. (c) $P^3 = P(I - P) = P - P^2 = P - (I - P) = 2P - 1$

$P^4 = P(2P - I) = 2(I - P) - P = 2I - 3P$

$P^5 = P(2I - 3P) = 2P - 3(IP) = 5P - 3I$

$P^6 = P(5P - 3I) = 5(I - P) - 3P = 5I - 8P$

So $n = 6$

4. (b) $(AB).(AB) = A(BA)B$

$= A^3B^2$

$(AB)(AB)(AB) = A^7B^3$

so $(AB)^n = A^{2^n - 1}B^n$

so $k = 2^{10} = 1 = 1023$

5. (a) Here $a_{ij} + a_{ji} = 0 \Rightarrow A^T = -A$

and $b_{ij} - b_{ji} = 0 \Rightarrow B^T = B$

and A, B are 3×3 matrices,

Hence $|A| = 0, \Rightarrow |A^4B^3| = 0$

$\Rightarrow A^4B^3$ is singular matrix.

6. (d) By observation its obvious that R_1 and R_2 are interchanged and R_3 of R.H.S -2 times R_3 of L.H.S.

i.e., $R_1 \to R_2, R_3 \to -2R_3 \Rightarrow A = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -2 \end{pmatrix}$

7. (b) $AB = \lambda B$, where λ is non-zero scalar.

$$\begin{bmatrix} -5x - 8y - 7 \\ 3x + 5y + 4 \\ 2x + 3y + 3 \end{bmatrix} = \lambda \begin{bmatrix} x \\ y \\ 2 \end{bmatrix}$$

i.e., $-5x - 8y - 7 = \lambda x$

$3x + 5y + 4 = \lambda y$

$2x + 3y + 3 = 2\lambda$

Adding $0 = \lambda(x + y + 2)$

$\lambda \neq 0 \Rightarrow x + y + 2 = 0$

$x + y = -2$

8. (d) Clearly option (d) satisfies the given conditions.

9. (d) As $a_{ij} = (i^2 + j^2 - ij)(j - i)$

$a_{ji} = (j^2 + i^2 - ji)(i - j) = -a_{ij}$

$\Rightarrow A$ is skew symmetric $\Rightarrow T_r(A) = 0$.

Also $|A| = 0$.

10. (a) $(A^3 + B^3 + C^3)(A - B) = A^4 - A^3B + B^3A - B^4 + C^3A - C^3B = 0$

$\Rightarrow \quad |(A^3 + B^3 + C^3)(A - B)| = 0$

$\Rightarrow \quad |(A^3 + B^3 + C^3)| = 0$, since $|(A - B)| \neq 0$.

11. (c) $(A + B)(A^2 - AB - B^2) = A^3 - A^2B - AB^2 + BA^2 - BAB - B^3$

$= A^3 - B^3 - A^2B - AB^2 - ABA + AB^2$ $(\because AB = -BA)$

$= A^3 - B^3 - A^2B + A^2B$

$= A^3 - B^3$

12. (d) $(XY)^T = Y^TX^T$

$Y^T = (ABC - CBA)^T$

$= C^TB^TA^T - A^TB^TC^T$

$= -CBA + ABC = Y$

$X^T = (ABC + CBA)^T$

$= C^TB^TA^T + A^TB^TC^T$

$= -CBA - ABC = -X$

13. (b) $(A^2 + P^2)(A - P) = A^3 - A^2P + P^2A - P^3$

$= (A^3 - P^3) + (P^2A - A^2P)$

$= 0$

$\therefore \quad |(A^2 + P^2)(A - P)| = 0$

$\therefore \quad |A^2 + P^2| = 0 \quad (\because |A - P| \neq 0)$

DPP 9.2

Single Correct Answer Type

1. (d) Given $AB = A$ and $BA = B$

$\Rightarrow \quad A^2 = A$

$B^2 = B$

$\Rightarrow \quad A^n = A$

$B^n = B$

$\Rightarrow \quad (A^{2010} + B^{2010})^{2011} = (A + B)^{2011}$

Now $(A + B)^2 = A^2 + B^2 + AB + BA$

$= 2(A + B)$

$\Rightarrow \quad (A + B)^k = 2^k(A + B)$

$\Rightarrow \quad (A^{2010} + B^{2010})^{2011} = (A + B)^{2011} = 2^{2011}(A + B)$

2. (c) $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$A + A^T = I$

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$$\Rightarrow \begin{pmatrix} a & b \\ c & d \end{pmatrix}+\begin{pmatrix} a & c \\ b & d \end{pmatrix}=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\Rightarrow 2a = 1, b + c = 0, 2d = 1$$

$$\Rightarrow a=\frac{1}{2}, c=-b, d=\frac{1}{2}$$

$$\Rightarrow A=\begin{pmatrix} \frac{1}{2} & b \\ -b & \frac{1}{2} \end{pmatrix}$$

Now $AA^T = I$

$$\Rightarrow \begin{pmatrix} \frac{1}{2} & b \\ -b & \frac{1}{2} \end{pmatrix}\begin{pmatrix} \frac{1}{2} & -b \\ b & \frac{1}{2} \end{pmatrix}=I$$

$$\Rightarrow \begin{pmatrix} \frac{1}{4}+b^2 & 0 \\ 0 & b^2+\frac{1}{4} \end{pmatrix}=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\Rightarrow b^2+\frac{1}{4}=1\Rightarrow b=\pm\frac{\sqrt{3}}{2}$$

$$\therefore A=\begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix} \text{ and } \begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$$

No. of matrices = 2

3. (c) Since A and B are orthogonal, $\det A = -1$, $\det B = 1$, or the other way round

Now $A^T(A + B)B^T = (A + B)^T$

We have on taking determinants on both sides

$(\det A)\ \det(A + B)\ \det B = \det(A + B)$

$\Rightarrow -\det(A + B) = \det(A + B)$

$\Rightarrow \det(A + B) = 0$

4. (d) $A=\begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix}$

$\therefore AA^T = I$ (i)

Now, $C = ABA^T$

$\Rightarrow A^TC = BA^T$ (ii)

Now $A^TC^nA = A^TCC^{n-1}A = BA^TC^{n-1}A$ (from (ii))

$= BA^TCC^{n-2}A = B^2A^TC^{n-2}A = \ldots$

$= B^{n-1}A^TCA = B^{n-1}BA^TA = B^n = \begin{bmatrix} 1 & 0 \\ -n & 1 \end{bmatrix}$

5. (c) Let $X=\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$,

$$(x_1 \quad x_2 \quad x_3)\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}=0$$

$a_{11}x_1^2 + a_{22}x_2^2 + a_{33}x_3^2 + (a_{12} + a_{21})x_1x_2 + (a_{13} + a_{31})x_1x_3 +$

$(a_{23} + a_{32})x_2x_3 = 0$

It is true for every x_1, x_2, x_3,

then $a_{11} = a_{22} = a_{33} = 0$, $a_{12} + a_{21} = 0$, $a_{13} + a_{31} = 0$, $a_{23} + a_{32} = 0$

$\therefore A$ is a skew symmetric matrix

$a_{21} = -2008$

$a_{31} = -2010$

$a_{32} = 2012$

6. (d) $A^6 = I \Rightarrow BA^6 = B$

$\Rightarrow (BA)\,A^5 = B$

$\Rightarrow AB^2\,A^5 = B$

$\Rightarrow AB\,(AB^2)\,A^4 = B$

$\Rightarrow A^2B^4\,A^4 = B$

Proceeding like this we get

$A^6B^{64} = B \Rightarrow B^{64} = B$

$\Rightarrow B^{63} = I$

$\Rightarrow k = 63$

7. (d) Let $A=\begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \end{bmatrix}$, $B=\begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \\ b_5 & b_6 \end{bmatrix}$

$$\text{So, } \det(BA) = \begin{vmatrix} b_1a_1 + b_2a_4 & b_1a_2 + b_2a_5 & b_1a_3 + b_2a_6 \\ b_3a_1 + b_4a_4 & b_3a_2 + b_4a_5 & b_3a_3 + b_4a_6 \\ b_5a_1 + b_6a_4 & b_5a_2 + b_6a_5 & b_5a_3 + b_6a_6 \end{vmatrix}$$

$$= \begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ 0 & 0 & 0 \end{vmatrix}\times\begin{vmatrix} b_1 & b_2 & 0 \\ b_3 & b_4 & 0 \\ b_5 & b_6 & 0 \end{vmatrix} = 0 \text{ (column by row)}$$

8. (b) Let $A=\begin{bmatrix} a & b & c \\ p & q & r \\ x & y & z \end{bmatrix}$

Given $a + b + c = 1$

$p + q + r = 1$

$x + y + z = 1$

$$\Rightarrow A^2=\begin{bmatrix} a & b & c \\ p & q & r \\ x & y & z \end{bmatrix}\begin{bmatrix} a & b & c \\ p & q & r \\ x & y & z \end{bmatrix}$$

$$=\begin{bmatrix} a^2 + bp + cx & ab + bq + cy & ac + br + cz \\ pa + qp + rx & pb + q^2 + ry & pc + qr + rz \\ xa + yp + zx & xb + yq + zy & xc + yr + z^2 \end{bmatrix}$$

Sum of elements of

$R_1 = a^2 + bp + cx + ab + bq + cy + ac + br + cz$

$= a(a + b + c) + b(p + q + r) + c(x + y + z)$

$= a + b + c = 1$

Similarly sum of elements of

$R_2 = p(a + b + c) + q(p + q + r) + r(x + y + z)$

$= p + q + r = 1$

$R_3 = x(a + b + c) + y(p + q + r) + z(x + y + z)$

$= x + y + z = 1$

$\therefore$ sum of elements of A^2 is 3.

9. (a) Given $AB + A + B = O$

$\Rightarrow AB + A + B + I = I$

$\Rightarrow A(B + I) + (B + I) = I$

$\Rightarrow (A + I)(B + I) = I$

$\Rightarrow (A + I)$ and $(B + I)$ are inverse of each other

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$\Rightarrow (A+I)(B+I) = (B+I)(A+I)$

$\Rightarrow AB = BA$

Thus A and B are commutative

$\Rightarrow (A+B)^2 = A^2 + 2AB + B^2$

10. (d) $(A+B)^2 = A^2 + B^2$

$\Rightarrow AB + BA = O$

$\Rightarrow AB = -BA$

$\Rightarrow |AB| = |-BA|$

$\Rightarrow |A||B| = -|B||A|$ (A and B are odd ordered matrices)

$\Rightarrow |B| = -|B|$ ($|A| = 2$)

$\Rightarrow |B| = 0$

Multiple Correct Answers Type

11. (a, b, c)

$$A^2 = \begin{bmatrix}1&0&0\\1&0&1\\0&1&0\end{bmatrix}\begin{bmatrix}1&0&0\\1&0&1\\0&1&0\end{bmatrix} = \begin{bmatrix}1&0&0\\1&1&0\\1&0&1\end{bmatrix}$$

$$A^3 = \begin{bmatrix}1&0&0\\1&1&0\\1&0&1\end{bmatrix}\begin{bmatrix}1&0&0\\1&1&0\\1&0&1\end{bmatrix} = \begin{bmatrix}1&0&0\\2&0&1\\1&1&0\end{bmatrix}$$

$$A^3 - A^2 = \begin{bmatrix}0&0&0\\1&-1&1\\0&1&-1\end{bmatrix} \text{ and } A - I = \begin{bmatrix}0&0&0\\1&-1&1\\0&1&-1\end{bmatrix}$$

$\Rightarrow A^3 - A^2 = A - I$ and $\det(A - I) = 0$

$\Rightarrow$ Det $|A^n - I|$ = Det $((A-I)(1 + A + A^2 + \ldots + A^{n-1}))$

= Det $(A-I)$ Det $(1 + A + A^2 + \ldots + A^{n-1}) = 0$

$A^3 - A^2 = A - I$ (i)

$\Rightarrow A^4 - A^3 = A^2 - A$ (ii)

$\Rightarrow A^5 - A^4 = A^3 - A^2 = A - I$ (Using (1))

If n is even $A^n - A^{n-1} = A^2 - A$ (iii)

If n is odd $A^n - A^{n-1} = A - I$ (iv)

Consider n is even

$\therefore A^n - A^{n-1} = A^2 - A$ (Using (iii))

$A^{n-1} - A^{n-2} = A - I$ (Using (iv))

On adding, we get

$A^n - A^{n-2} = A^2 - I$

$\Rightarrow A^n = A^{n-2} + A^2 - I$

$= (A^{n-4} + A^2 - I) + A^2 - I$

$= (A^{n-6} + A^2 - I) + 2(^2 - I)$

$= (A^2) + \dfrac{n-2}{2}(A^2 - I)$

$$A^n = \left(\frac{n}{2}\right)A^2 - \left(\frac{n-2}{2}\right)I$$

$\therefore A^{50} = 25A^2 - 24I$

$$= 25\begin{bmatrix}1&0&0\\1&1&0\\1&0&1\end{bmatrix} - 24\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$= \begin{bmatrix}1&0&0\\25&1&0\\25&0&1\end{bmatrix}$$

12. (a, c, d)

$$A = \begin{pmatrix}a&b\\c&d\end{pmatrix} \qquad a \neq b \neq c \neq d > 0$$

$$A + A^T = \begin{pmatrix}2a&b+c\\b+c&2d\end{pmatrix}$$

$|A + A^T| = 4ad - (b+c)^2 = 0 \Rightarrow b + c = 2\sqrt{ad}$

$\Rightarrow b + c = 2\sqrt{ad} > 2\sqrt{bc}$ (A.M > G.M)

$\Rightarrow ad > bc$

$\Rightarrow ad - bc > 0$ (as $a \neq b \neq c \neq d > 0$)

$\Rightarrow \det A > 0$

$$|A - A^T| = \left|\begin{pmatrix}0&b-c\\c-b&0\end{pmatrix}\right| = 0 + (b-c)^2 > 0$$

$$|AA^T| = |A||A^T| = |A|^2 = (\det A)^2 > 0$$

13. (a, d)

Let $X = \begin{bmatrix}a\\b\\c\end{bmatrix}$

$$\therefore \quad AX = \lambda X \Rightarrow \begin{bmatrix}8&-6&2\\-6&7&-4\\2&-4&3\end{bmatrix}\begin{bmatrix}a\\b\\c\end{bmatrix} = \begin{bmatrix}\lambda a\\\lambda b\\\lambda c\end{bmatrix}$$

$\Rightarrow (8 - \lambda)a - 6b + 2c = 0$

$-6a + (7 - \lambda)b - 4c = 0$

and $2a - 4b + (3 - \lambda)c = 0$

For non-zero solution $\begin{vmatrix}8-\lambda&-6&2\\-6&7-\lambda&-4\\2&-4&3-\lambda\end{vmatrix} = 0$

$\Rightarrow \lambda = 0, 3, 15$

14. (a, c)

$A + B = AB$

$\Rightarrow I - (A + B - AB) = I$

$\Rightarrow (I - A)(I - B) = I$

$\Rightarrow |I - A||I - B| = I$

$\Rightarrow |I - A|, |I - B|$ are non zero

Also $(I - B)(I - A) = I$

$\Rightarrow I - B - A + BA = I$

$\Rightarrow A + B = B + A$

$\Rightarrow AB = BA$

$\Rightarrow$ (a) and (c) are correct.

DPP 9.3

Single Correct Answer Type

1. (c) Adj $(4A) = 4^2$ Adj$(A) = 16$Adj (A)

$\Rightarrow |\text{Adj } 4A| = 16^3 |\text{Adj } A| = 16^3 \cdot 5^2$

2. (a) $AB = BA$

Pre and post multiplying both sides by A^{-1}

$\Rightarrow A^{-1}(AB)A^{-1} = A^{-1}(BA)A^{-1}$

$\Rightarrow (A^{-1}A)BA^{-1} = A^{-1}B(AA^{-1})$

$\Rightarrow BA^{-1} = A^{-1}B$

$\Rightarrow (BA^{-1})^T = (A^{-1})^T B^T = A^{-1}B$ (since A is symmetric, $\therefore A^{-1}$ is also symmetric)

Thus $(A^{-1}B)^T = A^{-1}B$

$\Rightarrow A^{-1}B$ is symmetric

$(A^{-1}B^{-1})^T = ((BA)^{-1})^T$

$= ((AB)^{-1})^T$

$= ((AB)^T)^{-1}$

$= (B^T A^T)^{-1}$

$= (BA)^{-1}$

$= A^{-1}B^{-1}$

$\Rightarrow A^{-1}B^{-1}$ is also symmetric

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3. (c) $|adj\ A^{-1}| = |A^{-1}|^2 = \dfrac{1}{|A|^2}$

$$|(adj\ A^{-1})^{-1}| = \frac{1}{|adj\ A^{-1}|} = |A|^2 = 2^2 = 4$$

4. (b) $|adj(adj(adj(adjA)))| = |A|^{16} = 4^8 \cdot 5^{16}$

$\Rightarrow \quad |A| = 10$

$\Rightarrow \quad x + y + z = 10$

Number of positive integral solutions is ${}^9C_2 = 36$

5. (d) Let $A = \begin{bmatrix} m & n \\ p & q \end{bmatrix}$, adj (a) $= \begin{bmatrix} q & -n \\ -p & m \end{bmatrix}$

Let $|A| = d = mq - np$

$$|A + d\ adjA| = \begin{vmatrix} m + qd & n(1-d) \\ p(1-d) & q + md \end{vmatrix} = 0$$

$\Rightarrow \quad mq + m^2d + q^2d + mqd^2 - np + 2npd - npd^2 = 0$

$\Rightarrow \quad (mq - np) + (mq - np)d^2 + m^2d + q^2d + 2mqd - 2d^2 = 0$

$\Rightarrow \quad (d + d^3 - 2d^2) + d(m^2 + q^2 + 2mq) = 0$

$\Rightarrow \quad d[(d-1)^2 + (m+q)^2] = 0 \Rightarrow d = 1, m + q = 0$

Now, $|A - d\ adj\ |A| = -(m+q)^2 + 4(mq - np) = 4d = 4$

6. (c) $B = (I - A)(I + A)^{-1}$

$\Rightarrow \quad B^T = (I + A^T)^{-1}(I - A^T)$

$= (I - A)^{-1}(I + A),$

$\Rightarrow \quad BB^T = (I - A)(I + A)^{-1}(I - A)^{-1}(I + A)$

$= (I - A)(I - A)^{-1}(I + A)^{-1}(I + A)$

$= I$

(As $(I - A).(I + A) = (I + A)(I - A)$)

7. (a) Here $|A| = 1$

$\Rightarrow \quad$ Adj (Adj A) $= |A|^{3-2} \cdot A = A$

(Where trace of a matrix is the sum of the elements in the principal diagonal)

8. (c) $\dfrac{|adj\ B|}{|C|} = \dfrac{|adj(adj\ A)|}{|5A|} = \dfrac{|A|^{(3-1)^2}}{5^3\,|A|} = \dfrac{|A|^2}{125}$

Now $|A| = 5$

$\therefore \quad \dfrac{|adj\ B|}{|C|} = 1$

9. (c) $BA = AB^2$

$\Rightarrow \quad BA = AB^2$

$\Rightarrow \quad A = B^{-1}AB^2$

$\Rightarrow \quad A^2 = (B^{-1}AB^2)(B^{-1}AB^2)$

$= B^{-1}A(BA)B^2$

$= B^{-1}A\,AB^2B^2$

$= B^{-1}A^2B^4$

$\therefore \quad A^3 = B^{-1}A^3B^6$

10. (c) Given $A^2 = A$

$\Rightarrow \quad I = (I - 0.4A)(I - \alpha A)$

$= I - I\alpha A - 0.4AI + 0.4\alpha A^2$

$= I - A\alpha - 0.4A + 0.4\alpha A$

$= I - A(0.4 + \alpha) + 0.4\alpha A$

$\Rightarrow \quad 0.4\alpha = 0.4 + \alpha$

$\Rightarrow \quad \alpha = \dfrac{-3}{2}$

11. (a)

$(A(A + B)^{-1}B)^{-1}(AB)$

$= B^{-1}((A + B)^{-1})^{-1}A^{-1}(AB)$

$= B^{-1}(A + B)A^{-1}(AB)$

$= (B^{-1}A + I)A^{-1}(AB)$

$= (B^{-1}AA^{-1} + A^{-1})(AB)$

$= (B^{-1} + A^{-1})(AB)$

$= B^{-1}AB + A^{-1}AB$

$= B^{-1}BA + A^{-1}AB$

$= A + B$

Multiple Correct Answers Type

12. (b, c)

$3ABA^{-1} + A = 2A^{-1}BA$

$\Rightarrow \quad 3ABA^{-1} + A + 2A = 2A^{-1}BA + 2A$

$\Rightarrow \quad 3A(BA^{-1} + I) = 2(A^{-1}B + I)A$

$\Rightarrow \quad 3A(B + IA)A^{-1} = 2A^{-1}(B + AI)A$

$\Rightarrow \quad 3A(B + A)A^{-1} = 2A^{-1}(B + A)A$

Let $B + A = X$

$\Rightarrow \quad 3AXA^{-1} = 2A^{-1}XA$

$\Rightarrow \quad 3^n|A||X||A^{-1}| = 2^n|A^{-1}||X||A|$

$\Rightarrow \quad 3^n|X| = 2^n|X|$ (as $|A| \neq 0$)

$\Rightarrow \quad |X| = 0$ or $|A + B| = 0$ (i)

Let $M = ABA^{-1} - A^{-1}BA$

$\therefore \quad AM = A^2BA^{-1} - BA \Rightarrow BA = A^2BA^{-1} - AM$

Now $3ABA^{-1} + A = 2A^{-1}BA$

$= 2A^{-1}(A^2BA^{-1} - AM)$

$= 2ABA^{-1} - 2M$

$\Rightarrow \quad ABA^{-1} + A = -2M$

$\Rightarrow \quad A(BA^{-1} + I) = -2M$

$A(A + B)A^{-1} = -2M$

Taking determinants both sides we get

$|-2M| = |A||A + B||A^{-1}| = 0$

$\Rightarrow \quad |ABA^{-1} - AB^{-1}A| = 0$

13. (b, c)

Let $(I - AB)^{-1} = P$

$\Rightarrow \quad P(I - AB) = I$

$\Rightarrow \quad P - PAB = I$

$\Rightarrow \quad PB^{-1} - PA = B^{-1}$

$\Rightarrow \quad BPB^{-1} - BPA = I$

$\Rightarrow \quad BPB^{-1} = I + BPA$

Now $BPB^{-1} = B(I - AB)^{-1}B^{-1}$

$= B(B(I - AB))^{-1}$

$= (B^{-1})^{-1}(B(I - AB))^{-1}$

$= (B(I - AB)B^{-1})^{-1}$

$= ((B - BAB)B^{-1})^{-1}$

$= (I - BA)^{-1}$

14. (a, b, c)

$A(adj\ A) = |A|I = 4I$

$\therefore \quad |A| = 4$

$|adj\ A| = |A|^{3-1} = 16$

$adj\ KA = K^{3-1}\ adj\ A$

$|adj(adj\ A)| = |A|^{(3-1)^2} = |A|^4 = 256$

$\therefore \quad |adj\ KA| = (K^{3-1})^3\ |adj\ A|$

$\therefore \quad |adj\ 2A| = 2^6 \cdot 16$

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CHAPTER 10

DPP 10.1

Single Correct Answer Type

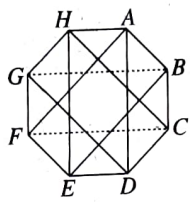
1. (d) As shown in figure here, four rectangles are formed.

$\therefore$ Number of favourable cases = 4

and total number of cases = ${}^8C_4$

$\therefore$ Required probability

$$= \frac{4}{{}^8C_4} = \frac{2}{35}$$



2. (b) Total no. of outcomes $=6^6$

Number of ways of choosing 4 other different numbers is ${}^6C_2$ and choosing 2 out of remaining 4 can be lone in ${}^4C_2$ ways.

Also number of ways of arranging 6 numbers of which 2 are alike and 2 are alike is $\frac{6!}{2!2!}$

$$\therefore \text{Required probability} = \frac{{}^6C_2 \times {}^4C_2 \times \frac{6}{2!2!}}{6^6} = \frac{26}{72}$$

3. (c) The total numbers of arrangements is $\frac{11!}{2!2!2!} = \frac{11!}{8}$

The number of arrangements in which C, E, H, I, S appear in that order ${}^{11}C_5 \frac{6!}{2!2!2!} = \frac{11!}{8 \cdot 5!}$

$$\therefore \text{Required Probability} = \frac{\frac{11!}{8 \cdot 5!}}{\frac{11!}{8}} = \frac{1}{120}$$

4. (a) Total number of ways of selecting two squares

$${}^{64}C_2 = \frac{1}{2}(64 \times 63) = 2016$$

If the first square happens to be any of the four corner ones, the second square can be chosen in 2 ways.

If the first square happens to be any of the 24 squares on the side of the chess board, the second square can be chosen in 3 ways.

If the first square happens to be any of the 36 remaining squares, the second square can be chosen in 4 ways.

Hence the required number of combinations

$$= \frac{1}{2}((4 \times 2) + (24 \times 3) + (36 \times 4)) = 112.$$

Therefore, the required probability = 1/18.

5. (c) Player A can win if A throws (1, 6) or (6, 1) and B throws ((1, 1), (2, 2), (3, 3), (4, 4), (5, 5) or (6, 6)). Thus the number of ways is 12.

Similarly the number of ways in which B can win is 12.

Total number of ways in which either A wins or B wins = 24

Thus the number of ways in which none of the two wins $= 6^4 - 24$.

$\therefore$ The required probability $= \frac{6^4 - 24}{6^4} = \frac{53}{54}$

6. (c) The expression $ax^4 + ax^2 + bx^3 + bx + x^2 + 1$

$= (x^2 + 1)(ax^2 + bx + 1)$

$\therefore x^2 + 1$ is positive for all real x.

For $ax^2 + bx + 1$ to be positive for all real x

$a > 0; b^2 - 4a < 0$

If $b = 1$, a can take 9 values from 1 to 9

$b = 2$, a can take 8 values from 2 to 9

$b = 3$, a can take 7 values from 3 to 9

$b = 4$, a can take 5 values from 5 to 9

$b = 5$, a can take 3 values from 7 to 9

b cannot take the values 6, 7, 8, 9.

$\therefore$ Number of favourable cases $= 9 + 8 + 7 + 5 + 3 = 32$

For each of the 9 values of a, there are 9 corresponding values for b.

$\therefore$ Number of exhaustive cases $= 9 \times 9 = 81$

$\therefore$ The required probability $= \frac{32}{81}$

7. (d) E_5 = the event of making words of 5 letters

E_6 = the event of making words of 6 letters

$P(E_6) = \frac{4! \times 3!}{6!}$, arrangement being done by considering all the vowels together for calculating $n(E_6)$

$P(E_5) = \frac{{}^3C_2 \cdot 4!.2! + {}^3C_2 \cdot 3!.3!}{{}^6P_5}$ {$\because$ there are 2 vowels, 3 consonants or 3 vowels, 2 consonants}

$\therefore$ The required probability

$$= P(E_6) + P(E_5) = \frac{4!.3! + 3.4!.2! + 3.3!.3!}{6!}$$

$$= \frac{144 + 144 + 108}{720} = \frac{396}{720} = \frac{11}{20}$$

8. (d) For symmetric matrix each place in upper triangle and leading diagonal can be filled in 7 ways. Then number of symmetric matrices are 7^6.

For skew symmetric matrix leading diagonal elements are zero. Upper triangle elements can be filled in 7^3 ways.

The required probability is $= \frac{7^6}{7^9} + \frac{7^3}{7^9} - \frac{1}{7^9}$ (as one matrix is common in both the cases)

9. (a) 1 2 3 4 5 6 7 8 9 10

1st drawn is 5, then 2nd drawn can be 1 only. If 1st is 6, then 2nd is 1 or 2 and so on.

$$\therefore \quad P(E) = \frac{1}{10}\left[\frac{1}{9} + \frac{2}{9} + \frac{3}{9} + \frac{4}{9} + \frac{5}{9} + \frac{6}{9}\right]$$

$$= \frac{1}{90}\left[\frac{6.7}{2}\right]$$

$$= \frac{7}{30}$$

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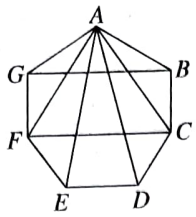
10. (d) ΔAGB, ΔAFC and ΔAED are isosceles



$$\therefore \quad P=\frac{{}^{7}C_1\times 3}{{}^{7}C_3}=\frac{7\times 3}{\frac{7\times 6\times 5}{3\times 2}}=\frac{3}{5}$$

11. (a) Let us first distribute 4 left gloves to four person in 4! ways.

If no one gets the corresponding right glove then no. of ways $= 4!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}\right)=9$

Then number of favorable ways $= 9\times 4!=216$

Total number of ways $4!\times 4!$

$$\therefore \text{ Required probability} = \frac{216}{4!\times 4!}=\frac{216}{576}=\frac{3}{8}$$

12. (b) For expression $\sin^{-1}(\sin x)+\cos^{-1}(\cos y)$ to be an integer, x should lie between $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ and y should lie between $[0,\pi]$

$x = 1$ and $y = 1, 2, 3$

$$\therefore \text{ Required probability} = \frac{3}{16}$$

13. (b) Each pre-image in set A can be assigned any one of the 5 images from set B

Hence total number of functions are $n(S)=5^4=625$

For strictly monotonic function either function is increasing or decreasing.

For which four different images are used, which can be selected in ${}^{5}C_4$ way.

Hence number of monotonic functions are $n(A)=2\cdot{}^{5}C_4=10$ (either increasing or decreasing)

$$\therefore \quad P(A)=\frac{10}{625}=\frac{2}{125}$$

14. (b) Each ball can be placed in 5 ways.

$\therefore$ Total number of ways, $n(S)=5^5$

2 empty boxes, can be selected in ${}^{5}C_2$ ways and 5 balls can be placed in the remaining 3 boxes in groups of 221 or 311 in

$$3!\left[\frac{5!}{2!2!2!}+\frac{5!}{3!2!}\right]=150 \text{ ways}$$

$\therefore$ Favorable cases $n(A)={}^{5}C_2\cdot 150$

$$\text{Hence } P(A)={}^{5}C_2\cdot\frac{150}{5^5}=\frac{60}{125}=\frac{12}{25}$$

15. (c) $n(S)=\frac{12!}{4!4!4!3!}3!=\frac{12!}{4!4!4!}$

$n(E)$ = the number of ways in which any one boy gets both the pens

$$={}^{3}C_1{}^{10}C_2\frac{8!}{4!4!2!}2!=3\times 45\frac{8!}{4!4!}$$

$$\therefore \quad P(E)=\frac{3\times 45\frac{8!}{4!4!}}{\frac{12!}{4!4!4!}}=\frac{2}{11}$$

$\therefore$ Required probability $= 1-\frac{2}{11}=\frac{9}{11}$

16. (d) Favourable cases for bijection is ${}^{100}C_9\times 9!$

For total cases

'B' is set of one element, no. of function -1

'B' is set of two element, no. of function -2^9

'B' is set of three element, no. of function -3^9

.....

.....

'B' = Y, No. of function $= 101^9$

$\therefore$ Total cases $= 1+{}^{100}C_1\cdot 2^9+{}^{100}C_2\cdot 3^9+\ldots+{}^{100}C_{100}\cdot 101^9$

$$=\sum_{r=1}^{101} r^9\cdot{}^{100}C_{r-1}$$

$$\therefore \text{ Required probability} = \frac{{}^{100}C_9\cdot 9!}{\sum_{r=1}^{101} r^9\cdot{}^{100}C_{r-1}}$$

17. (b) $\log_a b=\frac{\log b}{\log a}$

Let $b=2^m$ and $a=2^n$ where m and n denote the exponents on the base 2 in the given set then $\log_a b=\frac{m}{n}$

Therefore, $\log_a b$ is an integer only if n divides m.

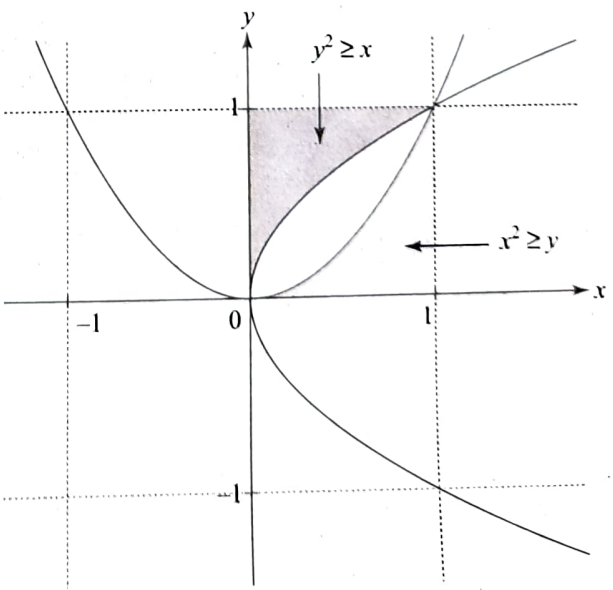
Now, total no. of ways m and n can be chosen $= 25\times 24=600$

For favorable cases

Let $n = 1$	So m can take values 1, 2, 3, 4, 5, 6, …, 24	$= 24$
if $n = 2$	m = 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24	$= 11$
$n = 3$	m = 6, 9, 12, 15, 18, 21, 24	$= 7$
$n = 4$	m = 8, 12, 16, 20, 24	$= 5$
$n = 5$	m = 10, 15, 20, 25	$= 4$
$n = 6$	m = 12, 18, 24	$= 3$
$n = 7$	m = 14, 21	$= 2$
$n = 8$	m = 16, 24	$= 2$
$n = 9, 10, 11, 12$	$m = 1$ for each	$= 4$
		62

Hence, required probability $=\frac{62}{600}=\frac{31}{300}$

18. (a)



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$$P(A)=\frac{\int_0^1\sqrt{x}dx}{\int_0^1 dx}=\frac{2}{3} \text{ and } P(B)=\frac{\int_0^1\sqrt{y}dy}{\int_0^1 dy}=\frac{2}{3}$$

$$\therefore \quad P(A\cap B)=\frac{\int_0^1\sqrt{x}dx-\int_0^1 x^2dx}{\int_0^1 dx}=\frac{2}{3}-\frac{1}{3}=\frac{1}{3}$$

19. (d) $A\cap B\subseteq B$

$$P(A\cap B)\le P(B)$$

$$P(A\cap B)\le \frac{5}{8} \qquad \text{(i)}$$

Now $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$$P(A\cup B)\le 1$$

$$\Rightarrow \quad P(A)+P(B)-P(A\cap B)\le 1$$

$$\Rightarrow \quad P(A\cap B)\ge \frac{3}{4}+\frac{5}{8}-1$$

$$\text{i.e., } P(A\cap B)\ge \frac{3}{8} \qquad \text{(ii)}$$

From (i) and (ii)

$$\frac{3}{8}\le P(A\cap B)\le \frac{5}{8}$$

$$\therefore \quad a+b=\frac{3}{8}+\frac{5}{8}=1$$

20. (d) $P(A'\cap B')=0.6$

$$\Rightarrow \quad P(A\cup B)'=0.6$$

$$\Rightarrow \quad P(A\cup B)=0.4$$

$$\therefore \quad P(A\cap B' \text{ or } A'\cap B)$$

$$=P(A\cap B')+P(A'\cap B)$$

$$=P(A\cup B)-P(A\cap B)=0.4-0.3=0.1$$

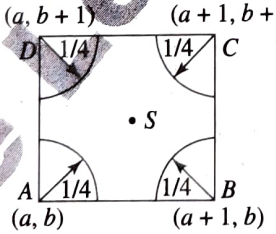
21. (a) Let event $A\rightarrow$ dealer will sell at least 20 TV sets and event $B\rightarrow$ dealer will sell less than 24 TV sets.

Now the event that dealer sells 20, 21, 22 or 23 TV sets is $A\cap B$

$\therefore P(A\cap B)=P(A)+P(B)-P(A\cup B)=0.45+0.74=0.19$

22. (a) Let S denote the set of points inside a square with corner (a, b), $(a, b+1)$, $(a+1, b)$, $(a+1, b+1)$

Let P denotes the set of points in S with distance less than 1/4 from any corner point.



Thus a coin whose centre falls in S will cover a point of X if and only if its centre falls in a point of P

$$\therefore \text{ Required probability } =\frac{\text{Area of } P}{\text{Area of } S}=\frac{\pi(1/4)^2}{1}=\frac{\pi}{16}\approx 0.2$$

Multiple Correct Answers Type

23. (a, d)

$P(A\cap B)=a$, $P(A)=a+d$, $P(B)=a+2d$ and $P(A\cup B)=a+3d$ also $a+d=d$

$$\Rightarrow \quad a=0$$

$$\Rightarrow \quad P(A\cap B)=0,\ P(A)=d,\ P(B)=2d$$

and $P(A\cup B)=3d$

24. (a, c, d)

$$P(A)=\frac{{}^5C_1 2^6}{2^{10}}=\frac{5}{16},\ P(B)=\frac{{}^5C_1 2^6}{2^{10}}=\frac{5}{16}$$

$$P(A\cap B)=\frac{{}^5C_2\times 2\times 2^3}{2^{10}}=\frac{5}{32}$$

$$\Rightarrow \quad P(A\cup B)=\frac{15}{32}$$

25. (a, b, c, d)

A, B are exhaustive events $\Rightarrow A\cup B=S$

$$\Rightarrow \quad P(A\cup B)=1$$

$$2P(B)=P(A)+P(A\cup B)=K+1$$

$$\Rightarrow \quad P(B)=\frac{K+1}{2}-P(A\cup B)-P(A)-PB$$

$$=1-K-\frac{(K+1)}{2}=\frac{2-2K-K-1}{2}$$

$$\therefore \quad P(A\cap B)=\frac{3K-1}{2}$$

$$P(A'\cup B')=P((A\cap B)')$$

$$=1-P(A\cap B)$$

$$=1-\frac{3K-1}{2}$$

$$=\frac{3(1-K)}{2}$$

26. (a, b, c)

Let the no. of blue marbles is a and no. of green marbles is b.

$$\therefore \quad \frac{ab}{{}^{a+b}C_2}=\frac{1}{2}$$

$$\Rightarrow \quad (a+b)(a+b-1)=4ab$$

$$\Rightarrow \quad b^2-(2a+1)b+a^2-a=0$$

but $b\in R\Rightarrow D=(2a+1)^2-4(a^2-a)=8a+1$

$\therefore 8a+1$ must be a perfect square.

Hence possible values of a are 3, 6, 10

27. (a, d)

From the given condition, it follows that ${}^nC_4$, ${}^nC_5$ and ${}^nC_6$ are in A.P.

i.e., $2.\ {}^nC_5={}^nC_4+{}^nC_6$

$$\frac{2n(n-1)(n-2)(n-3)(n-4)}{5!}$$

$$=\frac{n(n-1)(n-2)(n-3)}{4!}$$

$$+\frac{n(n-1)(n-2)(n-3)(n-4)(n-5)}{6!}$$

$$\Rightarrow \quad \frac{2(n-4)}{5}=1+\frac{(n-4)(n-5)}{5.6}$$

$$\Rightarrow \quad n^2-21n+98=0$$

$$\Rightarrow \quad (n-7)(n-14)=0$$

$$\Rightarrow \quad n=7,\ n=14$$

28. (b, c)

(a) is false since $A_1, A_2, \ldots, A_n$ may be overlapping.

(b) if $A_1, A_2, \ldots, A_n$ are disjoint and exhaustive both then $\sum P(A_i)=1$ if they are only exclusive then $\sum P(A_i)\le 1$.

If exhaustive then $\sum P(A_i)\ge 1$ (choice (c) follows)

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DPP 10.2

Single Correct Answer Type

1. (d) The probability of getting 2 or 4 or 5 in a single trial is

$$p=\frac{1}{6}+\frac{1}{6}+\frac{1}{6}=\frac{1}{2}$$

$$\therefore \quad q=\frac{1}{2}$$

The probability of getting 2 or 4 or 5 at most 15 times while throwing a die 31 times

$$={}^{31}C_0\cdot q^{31}+{}^{31}C_1\cdot pq^{30}+{}^{31}C_2\,p^2q^{29}+\ldots+{}^{31}C_{15}\,p^{15}q^{16}$$

$$=({}^{31}C_0+{}^{31}C_1+\ldots+{}^{31}C_{15})\frac{1}{2^{31}} \qquad \left(\because p=q=\frac{1}{2}\right)$$

$$=\frac{1}{2}({}^{31}C_0+{}^{31}C_1+\ldots+{}^{31}C_{31})\frac{1}{2^{31}} \qquad (\because {}^{31}C_{16}={}^{31}C_{15}, \text{etc.})$$

$$=\frac{1}{2}\cdot 2^{31}\cdot\frac{1}{2^{31}}=\frac{1}{2}$$

2. (a) Probability of disease is fatal = $p = 10\%$

$$p=\frac{10}{100}=\frac{1}{10}, q=\frac{9}{10}$$

Number of patients = 6, Number of die cases = 3

$\therefore$ Probability that 3 will die

$$={}^6C_3\left(\frac{1}{10}\right)^3\left(\frac{9}{10}\right)^3=1458\times10^{-5}$$

3. (b) No. of trials $n = 21$

Success in solving the problem

$$\therefore \quad p=P(A\cup B\cup C)=1-P(\bar{A})P(\bar{B})P(\bar{C})$$

$$=1-\frac{3}{4}\times\frac{4}{5}\times\frac{5}{6}=\frac{1}{2}$$

$$\therefore \quad q=\frac{1}{2}$$

$\therefore$ Required probability $P(X\geq 11)=\frac{1}{2}$

4. (b) $A = \{HH, HTHH, HTHTHH, \ldots\}$

or $B = \{TT, THTT, THTHTT, \ldots\}$

$$P(A)=\frac{1}{2^2}+\frac{1}{2^4}+\frac{1}{2^6}+\ldots\ldots=\frac{\frac{1}{4}}{1-\frac{1}{4}}=\frac{1}{3}=P(B)$$

$\therefore$ Required probability $=\frac{1}{3}+\frac{1}{3}=\frac{2}{3}$

5. (a) At event A he gets a number greater than 3

$\therefore$ $P(A) = 1/2$

At event B he gets 5 in last throw

$\therefore$ $P(B) = 1/6$

$\therefore$ Required probability is

$$P(E)=\frac{1}{6}+\frac{1}{2}\cdot\frac{1}{6}+\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{6}+\ldots\ldots\infty$$

$$=\frac{\frac{1}{6}}{1-\frac{1}{2}}=\frac{1}{3}$$

6. (c) $P(A\cup B)=P(A)+P(B)-P(A\cap B)$ (i)

When A and B are mutually exclusive, then $P(A\cap B)=0$

$\therefore$ $0.8 = 0.5 + p$

$\therefore$ $p = 0.3$

When A and B are independent $P(A\cap B)=P(A)\cdot P(B)$

$\therefore$ From (i), $0.8 = 0.5 + q - (0.5)\,q$

$$\therefore \quad 0.3=\frac{q}{2}$$

$$\Rightarrow \quad q=0.6$$

$$\Rightarrow \quad 2p=q$$

7. (d) $P(T) = p$; $P(H) = 1 - p$

$\therefore$ Required probability

$= P(T \text{ or } HHT \text{ or } HHHHT \text{ or } \ldots)$

$= p + (1-p)^2p + (1-p)^4p + \cdots$

$$=\frac{p}{1-(1-p)^2}$$

$$=\frac{p}{2p-p^2}=\frac{2}{3}$$

$$\Rightarrow \quad p=\frac{1}{2}$$

8. (c) $E : B_1\,B_2\,B_3\times\times\times$, where $\times$ means B or G

$$\therefore \quad P(E)=\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}=\frac{1}{8}=\frac{8}{64}$$

9. (b) ${}^4C_2\,p^2\,(1-p)^2={}^4C_3\,p^3\,(1-p)$

$$\Rightarrow \quad 6(1-p)=4p$$

$$\Rightarrow \quad p=\frac{3}{5}=0.6$$

10. (b) If A takes r shot then B will take more than r shots so required probabilities

$$=\Sigma P(A')^{r-1}\cdot P(A)[P(B')^r\cdot P(B)+P(B')^{r+1}\cdot P(B)+\ldots]$$

$$=\Sigma(P(A'))^{r-1}\cdot P(A)\cdot\frac{(P(B'))^r\,P(B)}{1-P(B')}$$

$$=\sum_{r=1}^{\infty}(P(A'))^{r-1}\cdot P(A)\cdot(P(B'))^r$$

$$=\frac{P(A)\cdot P(B')}{1-P(A')\cdot P(B')}=\frac{3/5\cdot 2/7}{1-2/5\cdot 2/7}=\frac{6}{31}$$

11. (c) Let event $S \to A$ is getting 1 or 2

Event $F \to B$ is getting 3, 4, 5, 6

$P(S) = P(1 \text{ or } 2) = 1/3$

$P(F) = P(3 \text{ or } 4 \text{ or } 5 \text{ or } 6) = 2/3$

$P(A \text{ wins}) = P[(S\,S \text{ or } S\,F\,S\,S \text{ or } S\,F\,S\,F\,S\,S \text{ or } \ldots) \text{ or } (F\,S\,S \text{ or } F\,S\,F\,S\,S \text{ or } \ldots)]$

$$=\frac{\frac{1}{9}}{1-\frac{2}{9}}+\frac{\frac{2}{27}}{1-\frac{2}{9}}$$

$$=\frac{1}{9}\times\frac{9}{7}+\frac{2}{27}\times\frac{9}{7}$$

$$=\frac{5}{21}$$

$$P\,(A \text{ winning}) = \frac{5}{21}$$

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Multiple Correct Answers Type

12. (a, d)

Clearly $P(A) = P(C)$

Event A is no boy or exactly one boy in family

$$\Rightarrow \quad P(A) = \left(\frac{1}{2}\right)^3 + {}^3C_1\left(\frac{1}{2}\right)^3 = \frac{1}{8} + \frac{3}{8} = \frac{1}{2}$$

Event B is 2 boy , 1 girl or 1 boy , 2 girls

$$P(B) = {}^3C_2\left(\frac{1}{2}\right)^3 + {}^3C_2\left(\frac{1}{2}\right)^3 = \frac{3}{4}$$

Event C is no girl or exactly one girl

$$P(C) = \left(\frac{1}{2}\right)^3 + {}^3C_1\left(\frac{1}{2}\right)^3 = \frac{1}{2}$$

Event $A \cap B$ is one boy and two girls

$$P(A \cap B) = {}^3C_1\left(\frac{1}{2}\right)^3 = 3/8$$

Event $B \cap C$ is one girl and two boys $P(B \cap C) = \frac{3}{8}$

$A \cap C \to \phi$

$\Rightarrow$ $A \cap B \cap C$ in also ϕ

$P(A \cap B) = \frac{3}{8} = P(A) \times P(B)$, so A and B are independent

neither $P(A \cap B \cap C) = P(A) \times P(B) \times P(B)$ nor $P(A \cap C) = P(A) \times P(C)$

So ABC is not independent.

13. (a, c)

We have $q = P(0H \text{ or } 2H \text{ or } 4H)$

$$\therefore \quad q = p^4 + {}^4C_2p^2(1-p)^2 + (1-p)^4$$
$$= 8p^4 - 16p^3 + 12p^2 - 4p + 1$$
$$= \frac{(2p-1)^4 + 1}{2}$$

Now put the values and check.

14. (a, b, c)

Let event E_1 is first digit is '2' $\Rightarrow$ 211 or 222

$$P(E_1) = \frac{2}{4} = \frac{1}{2}$$

Let event E_2 is second digit is '2' $\Rightarrow$ 121, 222

$$P(E_2) = \frac{1}{2}$$

Let event E_3 is third digit is '2' $\Rightarrow$ 222 and 112

$$P(E_3) = \frac{1}{2}$$

$(E_1 \cap E_2) = 222 \quad \Rightarrow P(E_1 \cap E_2) = \frac{1}{4} = P(E_1)\,P(E_2)$

Similarly E_2 and E_3, E_1 and E_3

Also $E_1 \cap E_2 \cap E_3 \to 222$

$$\Rightarrow \quad P(E_1 \cap E_2 \cap E_3) = \frac{1}{4} \neq P(E_1)\,P(E_2)\,P(E_3)$$

DPP 10.3

Single Correct Answer Type

1. (d) $P\left(\frac{B}{A}\right) = \frac{1}{2} \Rightarrow \frac{P(B \cap A)}{P(A)} = \frac{1}{2} \Rightarrow P(B \cap A) = \frac{1}{8}$

$$P\left(\frac{A}{B}\right) = \frac{1}{4} \Rightarrow \frac{P(A \cap B)}{P(B)} = \frac{1}{4} \Rightarrow P(B) = \frac{1}{2}$$

$$P(A \cap B) = \frac{1}{8} = P(A) \cdot P(B)$$

$\therefore$ Events A and B are independent.

Now, $P\left(\frac{A'}{B}\right) = \frac{P(A' \cap B)}{P(B)} = \frac{P(A')P(B)}{P(B)} = \frac{3}{4}$

and $P\left(\frac{B'}{A'}\right) = \frac{P(B' \cap A')}{P(A')} = \frac{P(B')P(A')}{P(A')} = \frac{1}{2}$

2. (c) Given $P(B') > P(A')$

$\Rightarrow \quad P(B) < P(A)$

$\Rightarrow \quad P(A) = P(A \cap B') + P(A \cap B)$
$> P(B)$
$= P(A' \cap B) + P(A \cap B)$

$\Rightarrow \quad P(A \cap B') > P(A' \cap B)$

$\Rightarrow (A)$ is false and so is (B)

$$P(B/A) = \frac{P(A \cap B)}{P(A)} < \frac{P(A \cap B)}{P(B)} = P(A/B)$$ $\because$ (c) is true.

3. (a) $P\left(\left(\frac{A^C}{B^C}\right)^C\right) = 1 - P\left(\frac{A^C}{B^C}\right)$

$$= 1 - \frac{P(A^C \cap B^C)}{P(B^C)}$$
$$= 1 - \frac{1 - P(A \cup B)}{1 - P(B)}$$
$$= 1 - \frac{1 - P(A) - P(B) + P(A \cap B)}{1 - P(B)}$$
$$= 1 - \frac{0.65}{0.75} = 1 - \frac{13}{15} = \frac{2}{15}$$

4. (c) Let C = the event of the selected number being composite,
E = the event of there being no remainder.

$$P(C) = \frac{n(C)}{n(S)} = \frac{15}{25} = \frac{3}{5}, P(\bar{C}) = \frac{2}{5}$$

$$P(E/C) = \frac{4}{15}, P(E/\bar{C}) = \frac{1}{10}$$

$$P(E) = P(C) \cdot P(E/C) + P(\bar{C}) \cdot P(E/\bar{C})$$
$$= \frac{3}{5} \cdot \frac{4}{15} + \frac{2}{5} \cdot \frac{1}{10} = \frac{4}{25} + \frac{1}{25} = \frac{1}{5} = 0.2$$

5. (a) $P(B/A \cup B') = \frac{P(B \cap (A \cup B'))}{P(A \cup B')}$

$$= \frac{P(A \cap B)}{P(A) + P(B') - P(A \cap B')}$$
$$= \frac{0.3}{0.7 + 0.5 - 0.4} = \frac{3}{8}$$

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6. (a) Let A denote the event that the runner succeeds exactly 3 times out of five and B denote the event that the runner succeeds on the first trial.

$$P(B/A)=\frac{P(B\cap A)}{P(A)}$$

But $P(B \cap A)$ = P(clearing succeeding in the first trial and exactly once in two other trials)

$= {}^4C_2\, p^2\,(1-p)^2) = 6p^3\,(1-p)^2$

and $P(A) = {}^5C_3\, p^3\,(1-p)^2 = 10p^3\,(1-p)^2$

Thus, $P(B/A) = \dfrac{6p^3(1-p)^2}{10p^3(1-p)^2} = \dfrac{3}{5}$

7. (b) Let A be the event of drawing a black ball.

A_1, the event of choosing the first box.

A_2, the event of choosing the second box.

$$\therefore \quad P\left(\frac{A_2}{A}\right)=\frac{P\left(\frac{A}{A_2}\right)\cdot P(A_2)}{P\left(\frac{A}{A_1}\right)\cdot P(A_1)+P\left(\frac{A}{A_2}\right)\cdot P(A_2)}$$

$$=\frac{\frac{4}{7}\cdot\frac{1}{2}}{\frac{3}{7}\cdot\frac{1}{2}+\frac{4}{7}\cdot\frac{1}{2}}=\frac{\frac{4}{7}}{1}=\frac{4}{7}$$

8. (d) Probability that they both solve correctly is $\frac{1}{8}\times\frac{1}{12}=\frac{1}{96}$

Probability that they both get wrong solution is $\frac{7}{8}\times\frac{11}{12}=\frac{77}{96}$

If E_1 be the event of both getting correct solution and E_2 be the event that both gets wrong solution.

Let E be even of both obtaining the same answer, then

$$P\left(\frac{E_1}{E}\right)=\frac{P(E_1)P\left(\frac{E}{E_1}\right)}{P(E_1)P\left(\frac{E}{E_1}\right)+P(E_2)P\left(\frac{E}{E_2}\right)}$$

$$=\frac{\frac{1}{96}\times 1}{\frac{1}{96}\times 1+\frac{77}{96}\times\frac{1}{1001}}=\frac{13}{14}$$

9. (c) $P(A) = \dfrac{3}{4}$; $P(B/A) = \dfrac{1}{4}$

$P(A/B) = \dfrac{2}{3}$

$P(B/A) = \dfrac{P(B\cap A)}{P(A)} = \dfrac{1}{4}$ (given)

$\therefore \quad P(B \cap A) = \dfrac{3}{4}\cdot\dfrac{1}{4}=\dfrac{3}{16}$

Now $P(A/B) = \dfrac{P(A\cap B)}{P(B)} = \dfrac{2}{3}$ (given)

$\therefore \quad P(B)=\dfrac{3}{2}\cdot\dfrac{3}{16}=\dfrac{9}{32}$

$\therefore \quad P(A\cup B)=\dfrac{3}{4}+\dfrac{9}{32}-\dfrac{3}{16}=\dfrac{24+9-6}{32}=\dfrac{27}{32}$

$\therefore \quad P(A^C\cap B^C) = 1 - P(A\cup B) = 1-\dfrac{27}{32}=\dfrac{5}{32}$

10. (b) Let A = event that balls are of different colors

B = event that one ball is red and other is white

$$P\left(\frac{B}{A}\right)=\frac{P(A\cap B)}{P(A)}$$

$$=\frac{\frac{{}^6C_1\times{}^3C_1}{{}^{13}C_2}}{\frac{{}^3C_1\times{}^6C_1+{}^3C_1\times{}^4C_1+{}^4C_1\times{}^6C_1}{{}^{13}C_2}}$$

$$=\frac{6\times 3}{6\times 3+3\times 4+4\times 6}=\frac{1}{3}$$

11. (d) Assuming the validity of Bayes' theorem,

$$P(B/A)=\frac{P(B)\cdot P(A/B)}{P(B)\cdot P(A/B)+P(C)\cdot P(A/C)}$$

$$=\frac{\frac{1}{2}\cdot\frac{1}{5}}{\frac{1}{2}\cdot\frac{1}{5}+\frac{1}{2}\cdot\frac{2}{7}}=\frac{7}{17}$$

12. (c) E: Event of getting on outcome (n_1, n_2, n_3) such that $i^{n_1}+i^{n_2}+i^{n_3}=1$

E_1: Event that fair die is thrown

E_2: Event that biased die is thrown

Fav. Cases : (n_1, n_2, n_3) must be an arrangement of {2, 4, 4} or {6, 4, 4} or {1, 3, 4}, {3, 5, 4}

$$P(E/E_1) = {}^3C_2\left(\frac{1}{6}\right)^3+{}^3C_2\left(\frac{1}{6}\right)^3+3!\left(\frac{1}{6}\right)^3+3!\left(\frac{1}{6}\right)^3$$

$$P(E/E_2) = {}^3C_2\cdot\left(\frac{2}{9}\right)^3+{}^3C_2\left(\frac{2}{9}\right)^3+3!\left(\frac{1}{9}\right)^2\left(\frac{2}{9}\right)+3!\left(\frac{1}{9}\right)^2\left(\frac{2}{9}\right)$$

$$P(E_1/E)=\frac{\frac{1}{2}\left[\frac{1}{12}\right]}{\frac{1}{2}\left[\left(\frac{1}{12}\right)+\frac{8}{81}\right]}=\frac{27}{59}$$

13. (d) Let $P(A) = P(B) = p$ (say)

$$P(B/A)=\frac{P(B\cap A)}{P(A)}$$

$\therefore \quad (0.8)p = P(B \cap A)$ (i)

$P(A \cup B) = p + p - P(A \cap B)$

$0.6 = 2p - p(0.8) = (1.2)p$

$\therefore \quad p=\dfrac{0.6}{1.2}=\dfrac{1}{2}$

$\therefore \quad P(A) = P(B) = \dfrac{1}{2}$

Now, $P[(A \cap \bar{B}) \cup (\bar{A} \cap B)]$

$= P(A) + P(B) - 2P(A \cap B)$

$= 1 - 2(0.8)\ \dfrac{1}{2} = 0.2 = \dfrac{1}{5}$

Multiple Correct Answers Type

14. (a, b, c, d)

$$P(A) = \frac{9}{10},$$

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$$P(B) = P(A \cap B) = \frac{9}{10} \times \frac{9}{10} = \frac{81}{100}$$

$$P\left(\frac{A}{\overline{B}}\right) = \frac{P(A \cap \overline{B})}{P(\overline{B})} = \frac{\frac{9}{10} \times \frac{1}{10}}{1 - \frac{81}{100}} = \frac{9}{19}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{9}{10}$$

15. (a, c, d)

$$P(E_2/E_1) = \frac{P(E_1 \cap E_2)}{P(E_1)}$$

$$\frac{1}{2} = \frac{P(E_1 \cap E_2)}{1/4}$$

$$\Rightarrow \quad P(E_1 \cap E_2) = \frac{1}{8} = P(E_2) \cdot P(E_1/E_2)$$

$$= P(E_2) \cdot \frac{1}{4}$$

$$\Rightarrow \quad P(E_2) = \frac{1}{2}$$

Since $P(E_1 \cap E_2) = \frac{1}{8} = P(E_1) \cdot P(E_2)$, events are independent

Also $P(E_1 \cup E_2) = \frac{1}{2} + \frac{1}{4} - \frac{1}{8} = \frac{5}{8}$

$\Rightarrow E_1$ and E_2 are not exhaustive.

16. (a, b, c, d)

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\frac{5}{8} = \frac{3}{8} + \frac{1}{2} - P(A \cap B)$$

$$\Rightarrow \quad P(A \cap B) = \frac{2}{8} = \frac{1}{4}$$

$$\text{Now } P(A^C/B) = \frac{P(A^C \cap B)}{P(B)} = \frac{P(B) - P(A \cap B)}{P(B)}$$

$$= 1 - \frac{1/4}{1/2} = \frac{1}{2}$$

$$2P(A/B^C) = \frac{2P(A \cap B^C)}{P(B^C)}$$

$$= \frac{2(P(A) - P(A \cap B))}{1 - P(B)}$$

$$= 4\left(\frac{3}{8} - \frac{2}{8}\right) = \frac{1}{2} = P(A^C/B)$$

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{1}{4} \cdot \frac{2}{1} = \frac{1}{2} = P(B)$$

Again $P(A^C/B^C) = \frac{P(A^C \cap B^C)}{P(B^C)} = \frac{1 - P(A \cup B)}{1 - P(B)}$

$$= 2\left(1 - \frac{5}{8}\right) = \frac{3}{4}$$

$$P(B/A^C) = \frac{P(B \cap A^C)}{1 - P(A)} = \frac{P(B) - P(A \cap B)}{5/8}$$

$$= \frac{\frac{1}{2} - \frac{1}{4}}{\frac{5}{8}} = \frac{1}{4} \cdot \frac{8}{5} = \frac{2}{5}$$

Hence $8\,P(A^C/B^C) = 15\,P(B/A^C)$

$$2P(A/B^C) = \frac{1}{2}$$

$$\Rightarrow \quad P(A/B^C) = \frac{1}{4} = P(A \cap B)$$

17. (b, d)

In the word "POSSIBILITY" there are 2 S's, 3 I's and 6 other different words.

$$P(A) = \frac{\frac{10!}{3!}}{\frac{11!}{3!\,2!}} = \frac{2}{11}$$

$$P(B) = \frac{\frac{9!}{2!}}{\frac{11!}{3!\,2!}} = \frac{3}{55}$$

$$P(A \cap B) = \frac{\frac{8!}{}}{\frac{11!}{3!\,2!}} = \frac{2}{165}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{2}{11} + \frac{3}{55} - \frac{2}{165} = \frac{30 + 9 - 2}{165} = \frac{37}{165}$$

$$P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{2}{165}}{\frac{2}{11}} = \frac{1}{15}$$

Comprehension Type

18. (c), 19. (a)

Let E_1 = Event that A wrote a plus sign

E_1 = Event that B wrote a plus sign

E = Event that the referee observes a plus sign.

Given $P(E_1) = \frac{1}{3} \Rightarrow P(E_2) = \frac{2}{3}$

$P(E/E_1)$ = Probability that none of B, C, D change sign + Probability that exactly two of B, C, D change sign

$$= \frac{1}{27} + 3\left(\frac{1}{3} \times \frac{2}{3} \times \frac{2}{3}\right) = \frac{13}{27}$$

$P(E/E_2)$ = Probability that all of B, C, D change sign + Probability that exactly one of B, C, D changes sign

$$= \frac{8}{27} + 3\left(\frac{2}{3} \times \frac{1}{3} \times \frac{1}{3}\right) = \frac{14}{27}$$

$$\therefore \quad P(E_1/E) = \frac{13}{41} \text{ (Using Bayes' Theorem)}$$

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